MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- Wednesday, 9/10/08
  - Finish up slides from last time
  - Predicates (Propositional Functions)
  - Quantifiers:
    - "For all"
    - "There exists"
- READING: 1.5-1.6
- EXERCISES:
  - pp. 50-51: 7-20
  - p. 59: 42-45, plus #60 for these problems
- ***Remember that Hand-in HW #1 is due tomorrow.***

Propositional Functions, aka Predicates

- A **function f: A ⟶ B** is a rule that, for each $x \in A$ (the ***domain***), assigns a unique value $f(x) \in B$ (the ***codomain***).
- A **propositional function**, or **predicate**, is a function **P: A ⟶ {T, F}.**
  - For each $x \in A$, P(x) is a proposition that is either true or false for that x.
  - The set A is called the **"domain of discourse" (or *universe* of discourse).**
- Example: P: **R** ⟶ {T,F}, P(x) = "$x^2 \geq x$."
  - What is P(-1), P(0), P(1), P(2/3), P(π)?

Quantifiers

- A **quantifier** is an added condition that makes a predicate into a proposition.
  - **$x^2 \geq x$**
    - Predicate, *not* proposition
  - **For every real number x, $x^2 \geq x$.**
    - Proposition. True? False?
  - **There is at least one real number x such that $x^2 \geq x$.**
    - Proposition. True? False?
  - **If n is any integer, then $n^2 \geq n$.**
    - Proposition. True? False?
- **Universal quantifier:**
  - **Symbol: ∀**
  - **Meaning: "For all"**
- **Existential quantifier:**
  - **Symbol: ∃**
  - **Meaning: "There exists"**
- **Without a quantifier, we say x is a "free" variable**
- **With a quantifier, we say x is a "bound" variable**
- **Write the 3 quantified predicates using ∀ and ∃.**

Proving quantified propositions

- **"For all $x \in S$, P(x)":**
  - Use the letter x represent *any* arbitrary element of S, prove that P(x) is true.
- **"There exists $x \in S$, P(x)":**
  - Find *at least one* member of S for which P(x) is true.
- Examples: How do we prove:
  - For all $x \in \mathbf{Z}$, $x^2 + x$ is an even number.
  - There exists $x \in \mathbf{Z}$ such that $x^2 + 1$ is an even number.

Wednesday, 9/10/08, Slide #4

Negating quantifiers

- **Let U = {students in this MC 215 class}, and for $X \in U$, let P(X) be the proposition: "X has brown eyes"**
- In words, **what is the statement:** $\forall X\ P(X)$
- What is the **negation** of that statement? I.e., what does the statement $\neg(\forall X\ P(X))$ mean?
- In words, **what is the statement:** $\exists\ X\ P(X)$
- What is the **negation** of that statement? I.e., what does the statement $\neg(\exists\ X\ P(X))$ mean?

Wednesday, 9/10/08, Slide #5

Generalized De Morgan's Laws for Logic

- **Theorem.** Suppose P is a predicate with domain of discussion U. Then
  - **(a)** $\neg(\forall x\ P(x)) \equiv \exists x\ (\neg P(x))$
  - **(b)** $\neg(\exists x\ P(x)) \equiv \forall x\ (\neg P(x))$
- **Example:** Suppose we want to *disprove* that P(x) is true for all x.
  - This is the same as proving the left side of (a).
  - What does the theorem tell us?
- **Same question for:** Suppose we want to *disprove* that P(x) is true for some value of x.
  - What does part (b) tell us?

Wednesday, 9/10/08, Slide #6

Nesting quantifiers

- Let U = **R**. Translate, and determine truth value:
  - **∀x ∃y (x + y = 0)**
  - **∃y ∀x (x + y = 0)**
  - **Do the two expressions have the same meaning?**
- Negate each of the above and apply De Morgan's Law multiple times, until you no longer need the negation symbol, ¬.
  - Again, what are the truth values?