

MC215: MATHEMATICAL REASONING  
AND DISCRETE STRUCTURES

- Wednesday, 9/17/08
  - Return, go over HW #1
  - From last time:
    - Indirect proof = proof of contrapositive
    - Proofs of equivalence = "if and only if" proofs
  - Proof by contradiction
- READING: 2.2
- EXERCISES:
  - pp. 86-87: 1, 2, 5, 8, 37

Wednesday, 9/17/08, Slide #1

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Proof by Contradiction

- A third method of showing  $A \Rightarrow B$  is true:
  - **ASSUME:**  $A \Rightarrow B$  is **false!** This means:  
**A is true and B is false.**
  - **PROVE:** Some clearly false statement must be true.
    - E.g., Assume  $A \Rightarrow B$  is false. Show  $0 = 1$ . Or show  $2n$  is odd. Or show the  $10 \in \emptyset$ . Anything you know is false.
    - Sometimes you might prove A is false – a contradiction since you assumed A was true. If you do this, check to see if your proof is really indirect:  $(\text{not } A) \Rightarrow (\text{not } B)$
  - Since your conclusion can't be true, your assumption must have been false, i.e.,  $A \Rightarrow B$  is **true!**

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Advice and Example

- Don't consider proof by contradiction until
  - You've first considered **DIRECT PROOF (first choice):**
    - Assume A, prove B
  - You've next considered **INDIRECT PROOF (second choice):**
    - Assume NOT A, prove NOT B
  - Only then, consider **PROOF BY CONTRADICTION (third choice):**
    - Assume A and (not B), prove this implies a known false statement.
- Prove: If  $n$  is an integer then  $n$  is not both even and odd.
  - Contradiction Proof: Suppose not!
    - So we assume  $n$  is an integer *and*  $n$  is both even and odd.
    - Show that this implies something *known* to be false

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## More examples

- Contradiction proofs are often useful to prove statements with *no explicitly stated hypothesis*:
  - The square root of 2 is irrational.
  - There are infinitely many prime numbers.
- Assume not: the statement is false.
- Prove that this implies something known to be false.

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## More advice (also see Summary on p. 89)

- Given a theorem “If  $P(x)$  then  $Q(x)$ ” to prove:
  - Make sure you understand what  $P(x)$  is saying and what all its terms mean
  - Do the same for  $Q(x)$
  - Maybe do a few examples, or draw some pictures, to convince yourself the theorem could be true (but examples are not a proof!)
  - Try first: Direct Proof;
  - Try second: Indirect Proof (contrapositive)
  - Try third: Proof by Contradiction

Wednesday, 9/17/08, Slide #5

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