MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- Friday, 9/19/08
  - From last time:
    - Proof by contradiction
  - Today:
    - Problem session on basic proof techniques
- READING: 2.2
- EXERCISES:
- HAND-IN HW #2 WILL BE POSTED ON OUR WEBSITE BY LATER TODAY!

Summary of three proof techniques for "$\forall x \in U$, If P(x) then Q(x)"

- DIRECT PROOF:
  - Assume: P(X)
  - Prove: Q(x)
- INDIRECT PROOF (PROOF OF CONTRAPOSITIVE)
  - Assume: $\neg$Q(x)
  - Prove: $\neg$P(x)
- PROOF BY CONTRADICTION
  - Assume: P(x) *and* $\neg$Q(x)
  - Prove: A contradiction occurs

Exercise 1

- **Theorem. The product of two rational numbers is a rational number.**
- 1. State this in If-then form, using letters x and y for the two numbers
- 2. Say what the assumption and conclusion are if you use:
  - (a) Direct Proof
  - (b) Proof of Contrapositive
  - (c) Proof by contradiction
- 3. Which technique seems best?
- 4. Prove the theorem using this technique.

Exercise 2

- **Theorem. The sum of a rational number and an irrational number is an irrational number.**
- 1. State this in If-then form, using letters x and y for the two numbers.
- 2. Say what the assumption and conclusion are if you use:
  - (a) Direct Proof
  - (b) Proof of Contrapositive
  - (c) Proof by contradiction
- 3. Which technique seems best?
- 4. Prove the theorem using this technique.

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Exercise 3

- **Theorem. For n ∈ Z, n is odd if and only if 5n + 6 is odd.**
- 1. State this as *two* if-then theorems, and then do steps 2-4 for each theorem.
- 2. Say what the assumption and conclusion are if you use:
  - (a) Direct Proof
  - (b) Proof of Contrapositive
  - (c) Proof by contradiction
- 3. Which technique seems best?
- 4. Prove the theorem using this technique.

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Exercise 4 (p. 86 #22)

- **Definition**. If $s_1, s_2, \ldots, s_n \in \mathbf{R}$, their ***average*** (or ***mean***) is given by the expression:

$$\frac{s_1 + s_2 + \cdots + s_n}{n}$$

- **Prove**: If the average of $s_1, s_2, \ldots, s_n = A$, then there is at least one value s_i with $s_i \leq A$.
- **Prove or disprove:** If the average of $s_1, s_2, \ldots, s_n = A$, then there is at least one value s_i with $s_i < A$.

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Exercise 5

- **Theorem. If *a, b* ∈ R, then the following statements are equivalent:**
  - **(1) *a* is less than *b***
  - **(2) The average of *a* and *b* is greater than *a***
  - **(3) The average of *a* and *b* is less than *b***
- Prove enough implications of the form "If (i) then (j)," where (i) and (j) are two of the three statement, to establish the truth of the theorem.
- Draw an "implication diagram" to demonstrate that you have proved a sufficient number of statements.