MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- **Monday, 10/13/08**
  - From last time:
    - **Countability** – see handout with more material plus exercises
  - Slides for Friday, 10/10/08:
    - **Sequences**
  - Today:
    - **Strings**
- **READING**: 3.2
- **EXERCISES**:
  - pp. 147: 116a-e, 120, 130, 131

Monday, 10/13/08, Slide #1

Countability and Sequences

- A set S is ***finite*** if and only there is a bijection from {1, 2, …, n} to S for some $n \in \mathbf{Z}^+$, or S = ∅.
- A set S is ***countably infinite*** if and only there is a bijection from $\mathbf{Z}^+$ to S.
- A set S is ***countable*** if and only it is either finite or countably infinite.
- Another way to view countability is in terms of *sequences:*
- **A set S is countable if and only if the elements of S can be listed as a finite or infinite sequence:**
  - **S finite: $S = \{s_1, s_2, \ldots, s_n\}$**
  - **S infinite: $S = \{s_1, s_2, \ldots, s_n, \ldots\}$**

Monday, 10/13/08, Slide #2

Finite Sequences vs. Strings

- A **string *s* over a set X** is a finite sequence of elements of X, except that:
  - X is restricted to be a **finite set;**
  - We think of X as an **alphabet;** and
  - We think of s as a **word**
- Example: Let X = {p, q, r, ∧, ∨ , ¬, (,)}
  - A sequence of elements from X:
    - **{p, ∧, q, ∨, ¬, r}**
  - A string over X:
    - **p∧q∨¬r**

Monday, 10/13/08, Slide #3

Definitions

- A ***string over X***, where X is a finite set, is a finite sequence of elements from X.
- The **null string** or **empty string**, denoted λ, is the string of 0 elements of X
- $\mathbf{X^*}$ denotes the set of all strings over X.
- $\mathbf{X^+}$ denotes the set of all non-null strings over X.
- The **length** of a string s, denoted |s| is the number of elements in s.

More definitions; string operations

- Shorthand: If $a \in X$, and $n \in \mathbf{N}$, then $\boldsymbol{a^n}$ denotes the string aaa…a (n a's in a row)
  - Example: If X = {a, b, c}, what string is $c^2a^3b^5$, and what is its length?
- A **substring** of a string s, is a string composed of *consecutive elements* of s.
  - Note difference from subsequence, in which elements need not be consecutive.
- If $\alpha, \beta \in X^*$ the string $\alpha\beta$, comprised of the elements of α followed by the elements of β, is called the **concatenation** of α and β.
- If $\alpha \in X^*$, then $\boldsymbol{\alpha^R}$ denotes α written in reverse.

Exercises

- **Example 1:** Consider these two functions, where A is an alphabet (i.e., A is a finite set):
  - **f:** $A^* \longrightarrow A^*$, $f(\alpha) = \alpha^R$
  - **g:** $A^* \times A^* \longrightarrow A^*$, $f(\alpha, \beta) = \alpha\beta$
- For each function: Is it one-to-one? Is it onto?
- **Example 2:** Prove that A^* is a countable set.