MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- **Monday, 12/1/08**
  - From last time:
    - **Linear Homogeneous Recurrence Relations**
  - Today:
    - **Combinatorics**
      - **Addition and Multiplication Principles**
      - **Inclusion-Exclusion Formula**
- **READING**: 6.1
- **EXERCISES**:
  - pp. 274-276: 4, 6, 10, 17, 19, 86-88

Combinatorics

- ***Combinatorics*** is the field of mathematics that studies ways to count complex quantities. E.g.,
  - How many comparisons in a complex sorting algorithm?
  - How many ways to schedule final exams at Skidmore?
  - How many different legal 8-digit user passwords?
- Also important for studying questions in ***probability and statistics***

Main Ideas from Introductory Combinatorics

- **The Addition and Multiplication Principles**
  - How to count 'ands' and 'ors'
- **The Inclusion-Exclusion Principle**
  - How to count unions of sets without over-counting (related to Addition Principle)
- **Permutations**
  - The number of ways to *arrange* objects
- **Combinations**
  - The number of ways to *combine* objects together
- **The Pigeonhole Principle**
  - A simple but surprisingly useful idea

The Multiplication Principle (also called The Product Rule)

- Suppose event A can occur n_A ways, and event B can occur in n_B ways, then the number of ways to do **A *and* B**, or equivalently, **A *then* B**, is the *product* $\mathbf{n_A \cdot n_B}$.
- If there are *three* events A, B, C, then the number of ways to do **A and B and C** is $\mathbf{n_A \cdot n_B \cdot n_C}$, and so on for more events.

Multiplication Principle Examples

- How many different 5-digit zip codes are possible? How many are possible with all digits different?
- Tonight I plan to go out to dinner at one of my 4 favorite restaurants, and then watch a recording of either The Daily Show, True Blood, or Fringe. How many options do I have?
- Canadian postal codes are of the form "LDL DLD", where L is a letter, and D is a digit (e.g. K1S 1C2). How many are there?

The Addition Principle (also called The Sum Rule)

- If event A can occur in n_A ways, and event B can occur in n_B ways, and there is **no overlap** between the events A and B, then the number of ways that the event **A *or* B** can occur is $\mathbf{n_A + n_B}$.
- In set language: If A and B are *disjoint* finite sets, then
$$|A \cup B| = |A| + |B|$$
- We say that A and B are ***"mutually exclusive"*** events if there is no way that both can occur simultaneously.

Addition Principle Examples

- A Skidmore committee is being formed. If there are 2200 students and 200 faculty,
  - How many ways to choose one member who is either faculty or student?
  - How many ways to choose two members, one student and one faculty?
- Tonight there are 8 movies to choose from at Wilton, 8 in Clifton Park, and 5 at Aimie's.
  - How many possibilities if all movies are different?
  - In fact, Wilton and Clifton Park have 7 movies in common, and neither has any in common with Aimie's. How many now?

Inclusion-Exclusion Formula

- **Counts unions when sets overlap:**
  $$|A \cup B| = |A| + |B| - |A \cap B|$$
- How many numbers from 0 to 100 are divisible by 6 or by 8?
- How many possible zip codes are there that begin with 1 or end with 1?
- Can we generalize the inclusion-exclusion formula to 3 sets, A, B, C?