MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- Monday, 9/8/08
  - Finish up discussion of from last time:
    - Rules of Precedence
    - Equivalent propositions
  - Converse, Inverse, Contrapositive
  - Biconditional ↔
  - Logical Identities
  - Proofs and rules of inference

- READING: 1.3-1.4
- EXERCISES: Do before next class, but not to hand in
  - pp. 30-31: 1-5, 70-71, 76
  - pp. 35-36: 1-3, 11-12

Alternate English Phrasing of Logical Operators

- **For each of the following, define simple propositions p and q, and write the sentence in terms of p, q, and the logical operators, $\wedge$, $\vee$, $\neg$, $\rightarrow$:**
- *1. Sean is a math major, but Laura is a computer science major.*
- *2. We're going to the beach today if it's sunny.*
- *3. Whenever I go swimming, I get water in my ear.*
- *4. In order to get a driver's license, it's necessary to pass a written exam. (Let p = One gets a driver's license, and q = One passes a written exam).*
- *5. You can get an A in CS 106 only if all your programs are error-free.*
- *6. It's not sufficient to show a driver's license to get through US Customs at the Canadian border.*

Necessary and sufficient conditions

- In the proposition $p \rightarrow q$,
  - p is called a ***sufficient condition*** for q
    - I.e., if we know that $p \rightarrow q$ is true, and also that p is true, then that's sufficient information to conclude that q is true
  - q is called a ***necessary condition*** for p
    - I.e., if we know that $p \rightarrow q$ is true, then in order that p is true, it's necessary that q be true

Converse, inverse, contrapositive

- If p and q are propositions, and we consider the implication $p \rightarrow q$.
- The ***converse*** of $p \rightarrow q$ is: $\mathbf{q \rightarrow p}$
  - The converse **swaps** p and q
- The ***inverse*** of $p \rightarrow q$ is: $\mathbf{\neg p \rightarrow \neg q}$
  - The inverse **negates** both p and q
- The ***contrapositive*** of $p \rightarrow q$ is: $\mathbf{\neg q \rightarrow \neg p}$
  - The contrapositive **swaps *and* negates** p and q

- **Example:** Phrase the following in the form $p \rightarrow q$, and then find the converse, inverse, and contrapositive of **"I carry an umbrella whenever it's raining."**
  - **Which of the four variations, if any, seem to be equivalent statements?**

The biconditional: $p \leftrightarrow q$

- The **biconditional, $p \leftrightarrow q$**, is true exactly when *p* and *q* have the ***same truth values***
- The symbol ↔ is read "if and only if"
- We also say, "*p* is a necessary and sufficient condition for *q.*"
- $p \leftrightarrow q$ means that *p* and *q* are equivalent propositions.
- **Examples:** Show that $p \leftrightarrow q$ is equivalent to:
  - a) $p \rightarrow q$ and $q \rightarrow p$
  - b) $p \rightarrow q$ and $\neg p \rightarrow \neg q$

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Logical Identities

- For each set identity we stated, there is a corresponding logical identity:
  - Replace sets ***A, B, C,*** by propositions ***p, q, r***
  - Replace ∩ **by** ∧, replace ∪ **by** ∨, replace **complement by negation** (i.e., A^c by ¬ p)
- **Example:** The 1st distributive law for sets:
  - **A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)** becomes
  - **p ∧ (q ∨ r) = (p ∧ q) ∨ (p ∧ r)**
- **Example:** What about the 1st deMorgan's Law:

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$$

What is a proof?

- A **proof** is:
  - A **sequence of true statements** (the *hypotheses*), which
  - **In conjunction with accepted axioms,** e.g. rules of logic, rules of arithmetic, rules of set theory, etc.,
  - Imply that another statement, ***a conclusion,* must be true**.
- **Example:** Suppose that
  - We *know* that p is true, and
  - We *know* that $p \rightarrow q$ true
  - It follows from definition of "$\rightarrow$" that q is true.
  - Written briefly as in text:
    - $\therefore$ means "therefore"

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

Rules of inference

- Rules that let us combine hypotheses to draw conclusions:
  - We used "modus ponens" in last example.

Rule of Inference	Name	Rule of Inference	Name
$\begin{array}{l} p \to q \\ \underline{p} \\ \therefore q \end{array}$	Modus Ponens	$\begin{array}{l} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$	Conjunction
$\begin{array}{l} p \to q \\ \underline{\neg q} \\ \therefore \neg p \end{array}$	Modus Tollens	$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$	Hypothetical syllogism
$\begin{array}{l} \underline{p} \\ \therefore p \vee q \end{array}$	Addition	$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$	Disjunctive syllogism
$\begin{array}{l} \underline{p \wedge q} \\ \therefore p \end{array}$	Simplification		