

MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- Wednesday, 9/10/08
 - Finish up slides from last time
 - Predicates (Propositional Functions)
 - Quantifiers:
 - "For all"
 - "There exists"
- READING: 1.5-1.6
- EXERCISES:
 - pp. 50-51: 7-20
 - p. 59: 42-45, plus #60 for these problems
- ***Remember that Hand-in HW #1 is due tomorrow.***

Propositional Functions, aka Predicates

- A **function** $f: A \rightarrow B$ is a rule that, for each $x \in A$ (the *domain*), assigns a unique value $f(x) \in B$ (the *codomain*).
- A **propositional function, or predicate, is a function** $P: A \rightarrow \{T, F\}$.
 - For each $x \in A$, $P(x)$ is a proposition that is either true or false for that x .
 - The set A is called the “**domain of discourse**” (or *universe of discourse*).
- Example: $P: \mathbf{R} \rightarrow \{T, F\}$, $P(x) = “x^2 \geq x.”$
 - What is $P(-1)$, $P(0)$, $P(1)$, $P(2/3)$, $P(\pi)$?

Quantifiers

- A **quantifier** is an added condition that makes a predicate into a proposition.
 - $x^2 \geq x$
 - Predicate, *not* proposition
 - For every real number x , $x^2 \geq x$.
 - Proposition. True? False?
 - There is at least one real number x such that $x^2 \geq x$.
 - Proposition. True? False?
 - If n is any integer, then $n^2 \geq n$.
 - Proposition. True? False?
- **Universal quantifier:**
 - **Symbol:** \forall
 - **Meaning:** "For all"
- **Existential quantifier:**
 - **Symbol:** \exists
 - **Meaning:** "There exists"
- **Without a quantifier, we say x is a "free" variable**
- **With a quantifier, we say x is a "bound" variable**
- **Write the 3 quantified predicates using \forall and \exists .**

Proving quantified propositions

- **“For all $x \in S$, $P(x)$ ”:**
 - Use the letter x represent *any* arbitrary element of S , prove that $P(x)$ is true.
- **“There exists $x \in S$, $P(x)$ ”:**
 - Find *at least one* member of S for which $P(x)$ is true.
- Examples: How do we prove:
 - For all $x \in \mathbf{Z}$, $x^2 + x$ is an even number.
 - There exists $x \in \mathbf{Z}$ such that $x^2 + 1$ is an even number.

Negating quantifiers

- Let $U = \{\text{students in this MC 215 class}\}$, and for $X \in U$, let $P(X)$ be the proposition: "X has brown eyes"
- In words, **what is the statement:** $\forall X P(X)$
- What is the **negation** of that statement? I.e., what does the statement $\neg (\forall X P(X))$ mean?
- In words, **what is the statement:** $\exists X P(X)$
- What is the **negation** of that statement? I.e., what does the statement $\neg (\exists X P(X))$ mean?

Generalized De Morgan's Laws for Logic

- **Theorem.** Suppose P is a predicate with domain of discussion U . Then
 - (a) $\neg(\forall x P(x)) \equiv \exists x (\neg P(x))$
 - (b) $\neg(\exists x P(x)) \equiv \forall x (\neg P(x))$
- **Example:** Suppose we want to *disprove* that $P(x)$ is true for all x .
 - This is the same as proving the left side of (a).
 - What does the theorem tell us?
- **Same question for:** Suppose we want to *disprove* that $P(x)$ is true for some value of x .
 - What does part (b) tell us?

Nesting quantifiers

- Let $U = \mathbf{R}$. Translate, and determine truth value:
 - $\forall x \exists y (x + y = 0)$
 - $\exists y \forall x (x + y = 0)$
 - **Do the two expressions have the same meaning?**
- Negate each of the above and apply De Morgan's Law multiple times, until you no longer need the negation symbol, \neg .
 - **Again, what are the truth values?**