MC215: MATHEMATICAL REASONING AND DISCRETE STRUCTURES

- Wednesday, 9/17/08
  - Return, go over HW #1
  - From last time:
    - Indirect proof = proof of contrapositive
    - Proofs of equivalence = "if and only if" proofs
  - Proof by contradiction

- READING: 2.2
- EXERCISES:
  - pp. 86-87: 1, 2, 5, 8, 37

Proof by Contradiction

- A third method of showing A ⇒ B is true:
  - **ASSUME:** A ⇒ B is **false!** This means: **A is true *and* B is false.**
  - **PROVE:** Some clearly false statement must be true.
    - E.g., Assume A ⇒ B is false. Show 0 = 1. Or show 2n is odd. Or show the 10 ∈ ∅. Anything you know is false.
    - Sometimes you might prove A is false – a contradiction since you assumed A was true. If you do this, check to see if your proof is really indirect: (not A) ⇒ (not B)
  - Since your conclusion can't be true, your assumption must have been false, i.e., A ⇒ B is **true!**

Advice and Example

- Don't consider proof by contradiction until
  - You've first considered **DIRECT PROOF (*first choice):***
    - Assume A, prove B
  - You've next considered **INDIRECT PROOF *(second choice):***
    - Assume NOT A, prove NOT B
  - Only then, consider **PROOF BY CONTRADICTION *(third choice)*:**
    - Assume A and (not B), prove this implies a known false statement.
- Prove: If *n* is an integer then *n* is not both even and odd.
  - Contradiction Proof: <u>Suppose not!</u>
    - So we assume *n* is an integer *and n* is both even and odd.
    - Show that this implies something *known* to be false

More examples

- Contradiction proofs are often useful to prove statements with *no explicitly stated hypothesis:*
  - The square root of 2 is irrational.
  - There are infinitely many prime numbers.
- Assume not: the statement is false.
- Prove that this implies something known to be false.

More advice (also see Summary on p. 89)

- Given a theorem “If P(x) then Q(x)” to prove:
  - Make sure you understand what P(x) is saying and what all its terms mean
  - Do the same for Q(x)
  - Maybe do a few examples, or draw some pictures, to convince yourself the theorem could be true (but examples are not a proof!)
  - Try first: Direct Proof;
  - Try second: Indirect Proof (contrapositive)
  - Try third: Proof by Contradiction