MC302 GRAPH THEORY
Tuesday, 10/6/09

- **Today:**
  - Return, go over HW #2
  - Questions for Exam #1 on Thursday
  - From last time: Prove tree characterization theorem
- **Reading:**
  - Still 2.2
- **Problems for HW #3:**
  - pp. 56-57, #2.2.3 (be sure to justify your answer!) and #2.2.4
- **Exercises:**
  - pp. 56-57, 2.2.1 & 2.2.2
- **Other:**
  - **Extra office hour tomorrow, Wednesday, 2-3:30**

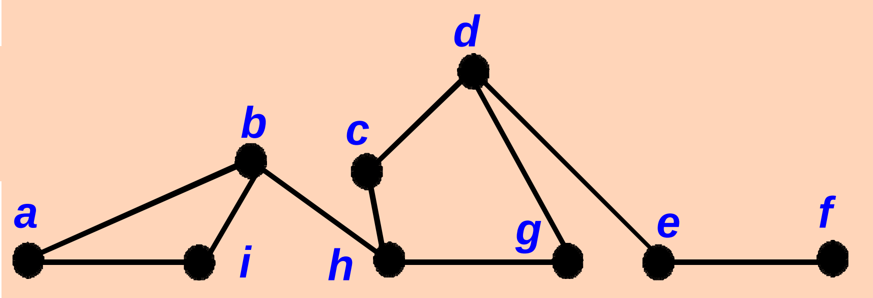
Edge deletion and connectedness

- **Theorem.** For any edge *e* of a graph G,
  $$\omega(G) \leq \omega(G - e) \leq \omega(G) + 1$$
  - In other words, deleting an edge from G either does not change the number of components *or* it increases the number by at most one.
- **Proof.** Let e = u-v, and let G\e have components $W_1, \ldots, W_k$ (i.e., $\omega(G - e) = k$).
  - **Case 1:** Vertices u and v are in same component of G\e. How many components does G have?
  - **Case 2:** Vertices u and v are in two different components of G\e. How many components does G have?

Bridges and Cycles

- **Definition.** A ***bridge (or cut edge)*** is an edge *e* of G with $\omega(G \setminus e) = \omega(G) + 1$.
  - *Warning: "Bridge"* has a second meaning in graph theory.

Which edges are bridges?



- **Theorem.** An edge *e* of G is a bridge *if and only if* it is not part of any cycle.
  - **Proof?** (Easier to do the contrapositive)

- **Theorem.** A graph G is a tree *if and only if* G is connected and every edge of G is a bridge.

Edge bounds for graphs

- **Theorem.** If *G* is a connected graph with *n* vertices, then *G* has at least *n-1* edges.
  - **Proof.** Use induction on the number of edges of *G* that are not bridges.
- **Corollary.** If *G* has fewer than *n-1* edges, then *G* is not connected.
- **Corollary.** If *G* is a graph with *n* vertices and *k* components, then *G* has at least *n-k* edges.
  - **Proof 1.** Induction on *k.*
  - **Proof 2.** Apply theorem to each component, and add up results.
- **Corollary.** $\omega(G) \geq |V(G)| - |E(G)|$