1. For the following questions, only a simple answer is required…and no computation should be needed. [5 pts]
   a. $\sum(X - \mu) = 0$
   b. Suppose that a sample standard deviation (s) is 10. Next, suppose that you add 5 to every score in the sample. The standard deviation of the sample would now be: 10
   c. Suppose that a sample standard deviation (s) is 10. Next, suppose that you convert every score in the sample to a z-score. The standard deviation of this distribution would be: 1
   d. Suppose that the mean of a sample of $n = 25$ is $M = 5$. Generally speaking, if you add another score (new $n = 26$), the $SS$ would increase. However, there is one score you could add to the sample without increasing the $SS$. That would be the mean (5 in this case).
   e. Researchers typically hope to achieve power of roughly .80. Essentially, in so doing, they are saying that they are willing to tolerate a Type II error rate of 20%.

2. Given that SAT math (SAT-M) scores are normally distributed with $\mu = 500$ and $\sigma = 100$, answer the following questions: [10 pts]
   a. What proportion (or percentage) of people would have SAT-M scores between 550 and 650?

   $z(550) = .5$ and $z(650) = 1.5$, so proportion is $0.3085 - 0.0668 = 0.2417$

   b. What SAT-M scores would determine the middle 50% of the distribution? (In other words, 25% above and 25% below the mean.)

   $z = \pm .67$, so 433 and 567.

   c. What proportion of groups of 25 students taking the SAT-M exam would have a mean greater than 400?

   $z(400) = -5$, so about 0.99999999

   d. Suppose that you wanted to translate individual SAT-M scores into a distribution with $\mu = 10$ and $\sigma = 1$. Under this new scoring system, what would an “old” SAT-M score of 600 translate into?

   $z(600) = 1$, so for the new scoring system with $\mu = 10$ and $\sigma = 1$, it would be 11.

3. Suppose that you take a random sample of $n = 9$ students on their way to an 8AM class in TLC and ask them for their GPAs. Their data are seen below. What is your best estimate of the $\mu$ and $\sigma$ for the population from which this sample was drawn? [10 pts]

   GPA | GPA²
   --- | ---
   3.5 | 12.25
   3.0 | 9.00
   3.8 | 14.44
   2.9 | 8.41
   3.7 | 13.69
   3.5 | 12.25
   3.1 | 9.61
   3.0 | 9.00
   3.2 | 10.24
   **Sum** | **98.89**
4. Given the sample of GPAs obtained above, and assuming a normal distribution of GPAs, how likely is it that your sample was drawn from a population with \( \mu = 3.3 \)? [10 pts]

\[ H_0: \mu = 3.3 \quad H_1: \mu \neq 3.3 \quad t_{\text{crit}}(8) = 2.306 \]

\[
t = \frac{3.3 - 3.3}{0.33/\sqrt{9}} = 0
\]

Decision: Retain \( H_0 \) because \( t_{\text{obt}} < t_{\text{crit}} \), so the sample could have come from a population with \( \mu = 3.3 \).

5. More miscellaneous questions

a. In a positively skewed distribution, what is the relationship between the mode, median, and mean? [2 pts]

Mode < Median < Mean

b. In class, we talked about the poor woman who was pregnant in San Diego. She argued that she had a very long gestation period (~308 days!!!). Essentially, she is claiming that her gestation period was sampled from the normal population (\( \mu = 268 \) days and \( \sigma = 16 \) days). When faced with a situation such as this, a statistician has to make a decision. Given her gestation period, what decision would you make? Why? What kind of error might you be making with that decision? [3 pts]

\[ H_0: \mu = 268 \quad H_1: \mu \neq 268 \quad z_{\text{crit}} = 1.96 \]

\[
z = \frac{308 - 268}{16} = 2.5
\]

Decision: Reject \( H_0 \) because \( z_{\text{obt}} > z_{\text{crit}} \), so the gestation period is unlikely to have come from a population with \( \mu = 268 \). However, you could be making a Type I error.

c. As the sample size approaches infinity, what happens to the value of \( t_{\text{critical}} \) and why? [2 pts]

It approaches 1.96 (\( z_{\text{crit}} \)) because the distribution of \( t \) becomes more normal as the sample size approaches infinity.

d. Interpret the PASW output below as completely as you can. (In other words, what is the statistic, what are the scores, how would you interpret the results, etc.) [3 pts]
\( H_0: \mu = 115 \quad H_1: \mu \neq 115 \)

Your measure is IQ scores, and you would retain \( H_0 \) because \( t(22) = 1.603, p = .123 \). (In other words, \( p > .05 \).)

6. When I asked about a way to detect a small effect size, Emily Bruschi answered that you would need a large sample size (\( n \)). If you recall, I responded that her response was sort of correct, but that she had skipped a step. That is, in order to detect a small effect size, you need to have more something (which might be achieved by a larger \( n \)). What is something? And how is it that Emily’s response would actually lead one to better detect a small effect size (by providing more something)? (A rough figure might help to explain your answer.) [5 pts]

The something, of course, is power. That is, when effect size is small, you need more power in order to detect that effect. The way that the larger sample size aids in increasing power is by reducing the variability in the sampling distribution of the mean. That is, as sample size increases, the standard error decreases. With a smaller standard error, the distributions would overlap less, even though the separation between the distributions is unchanged.

\[ \begin{array}{cccc}
N & \text{Mean} & \text{Std. Deviation} & \text{Std. Error} \\
22 & 123.1478 & 29.9680 & 5.20620 \\
\end{array} \]

\[ \begin{array}{cccccc}
\text{df} & \text{Sig. (2-tailed)} & \text{Mean Difference} & \text{95\% Confidence Interval of the Difference} \\
22 & .123 & 8.3473 & -2.4493 & 19.1450 \\
\end{array} \]