

1. In a short sentence, define power ($1 - \beta$). Then, tell me a strategy for making your statistic more powerful...being careful to articulate how that strategy works. [5 pts]

Power is the probability of correctly rejecting H_0 . That is, power represents the probability that you correctly decide that your sample mean was drawn from a population with a μ different from that of the μ proposed in H_0 . Later on we'll talk about all sorts of ways to make a study more powerful. At this point, you need to be able to articulate the important role of sample size (n). That is, you need to recognize that increasing the sample size makes your statistic more powerful. How it does so is by decreasing the standard error. As the standard error decreases, the critical values are pulled in ever closer to the mean proposed under H_0 . As a result, smaller differences from the hypothesized μ will lead to the rejection of H_0 .

2. Suppose that during impersonal social interactions (that is, with business or casual acquaintances) people in the United States maintain a mean social distance of $\mu = 4$ feet from the other individual. This distribution is normal and has $\sigma = 1.5$. A researcher examines whether or not this is true for other cultures. A random sample of $n = 8$ people of Middle Eastern culture is observed in an impersonal interaction. For this sample, the mean distance is 2.5 feet. Test the hypothesis that the Middle Eastern sample was drawn from a population with $\mu = 4$. [10 pts]

$$H_0: \mu = 4$$

$$H_1: \mu \neq 4$$

Decision Rule: If $|z| \geq 1.96$, reject H_0 .

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{8}} = .53$$

$$z = \frac{2.5 - 4}{.53} = \frac{-1.5}{.53} = -2.8$$

Because $|z| \geq 1.96$, I would reject H_0 and conclude that the sample was drawn from a population with $\mu < 4$.

3. Define the term "standard error." What is the formula for standard error? [5 pts]

The standard error is the standard deviation of the sampling distribution of the mean. The formula for the standard error is:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

4. Suppose that you take a random sample of $n = 9$ students on their way to an 8AM class in TLC and ask them for their GPAs. Their data are seen below. What is your best estimate of the μ and σ for the population from which this sample was drawn? [10 pts]

	Score	Score Squared
	3.4	11.56
	3.3	10.89
	3.0	9.0
	2.8	7.84
	3.6	12.96
	3.4	11.56
	3.5	12.25
	3.8	14.44
	3.2	10.24
Sum	30.0	100.74

$$\bar{X} = \hat{\mu} = \frac{\sum X}{n} = \frac{30}{9} = 3.33$$

$$SS = \sum X^2 - \frac{(\sum X)^2}{n} = 100.74 - 100 = .74$$

$$s = \hat{\sigma} = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{.74}{8}} = \sqrt{.0925} = .304$$

5. Given the sample of GPAs obtained above, and assuming a normal distribution of GPAs, how likely is it that your sample was drawn from a population with $\mu = 3.1$? [10 pts]

Because σ is not known, and you have to estimate it from the sample standard deviation, s , this problem requires that you use the t-test.

$$H_0: \mu = 3.1$$

$$H_1: \mu \neq 3.1$$

Decision Rule: if $|t_{\text{Obtained}}| \geq t_{\text{Critical}}$, reject H_0 .

$$t_{\text{Critical}}(8) = 2.306$$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{.3}{3} = .1$$

$$t = \frac{3.33 - 3.1}{.1} = \frac{.23}{.1} = 2.3$$

Although it's a close call, I would retain H_0 in this case. Of course, I would also presume that with more power, I might well be able to reject H_0 .

6. As you know, we can treat IQs as normally distributed with $\mu = 100$ and $\sigma = 15$. [15 pts]
a. What percentage of the population would have IQs between 80 and 90?

$$z = \frac{80 - 100}{15} = \frac{-20}{15} = -1.33$$

$$z = \frac{90 - 100}{15} = \frac{-10}{15} = -.67$$

.2514 - .0918 = .1596, so roughly 16% of the population would have IQs between 89 and 90.

- b. What percentage of the population would have IQs greater than 110?

$$z = \frac{110 - 100}{15} = \frac{10}{15} = .67$$

.2514, so roughly 25% of the population would have IQs ≥ 110 .

- c. What percentage of sample means from samples of size $n = 9$ would have IQs greater than 110?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{9}} = 5$$

$$z = \frac{110 - 100}{5} = 2$$

.0228, so roughly 2% of the samples of $n=9$ will have mean IQs ≥ 110 .

- d. What percentage of sample means from samples of size $n = 25$ would have IQs greater than 110?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{25}} = 3$$

$$z = \frac{110 - 100}{3} = 3.33$$

.0005, so roughly “no” samples of size $n = 25$ would have mean IQs ≥ 110 .

- e. Why are your answers to c and d different? What process does the difference reflect?

Because the standard error is smaller with larger sample sizes, it's much more difficult to get sample means of 110 from this population with a sample of 25 compared to a sample of 9.