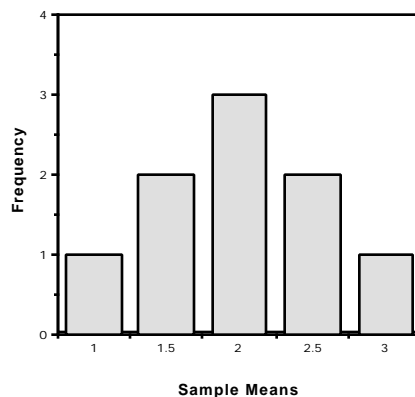


Sampling Distribution of the Mean

Many students find it difficult to grasp the concept of the sampling distribution. Here's an attempt to clarify the concept.

Think back to the handout on degrees of freedom. In that handout, I tried to show you how the denominator of the sample variance had to be $n-1$ in order to accurately estimate the population variance. In doing that exercise, you were actually creating a sampling distribution of the mean, so let's re-examine that exercise.

First, you were dealing with a very small (and unlikely) population: 1, 2, 3. That population has a mean (μ) of 2 and a variance (σ^2) of .67 and a standard deviation (σ) of .82. Taking every possible sample of $n=2$ from that population (sampling with replacement) gave us 9 sample means, which I have graphed below:



This distribution is actually the sampling distribution of the mean for samples of $n=2$. What is the mean of the sampling distribution? Of course, it is the population mean (2). What is the standard deviation of the sampling distribution (called the standard error of the mean)? Is it the population standard deviation? No, it is not. In fact, the standard error, which is the population standard deviation divided by the square root of the sample size

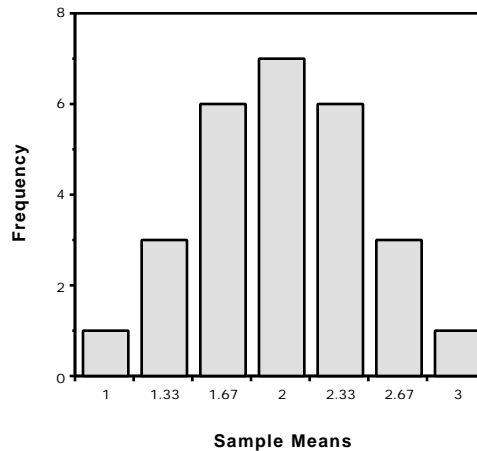
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

will almost always be smaller than the population standard deviation. In this case, the formula tells us that the standard error is .58. You could also compute the standard error for this distribution by actually computing the standard deviation of the 9 sample means (dividing the SS of the means by n , because there are n degrees of freedom — we aren't dealing with a sample). You would get the same answer.

Can you see why the standard error must be smaller than σ ? (Of course, you should be able to see why it must be so from the formula, but do you have a conceptual understanding of why this must be so?) One way to think about the issue is to ask yourself what happens to extreme scores in the sampling distribution. The greater the number of extreme scores, the greater the variability of a distribution, right? Okay, then ask yourself how often you could get a score of 3 in the population? You should see that a score of 3 would occur with a probability of .33 (1 in 3). How often would a mean of 3 occur in a sampling distribution of the mean from samples of $n=2$? Again, you should see that this is a less likely event, with a probability of .11 (1 in 9).

Here's another way to think about the issue. What would happen if we dealt with samples of $n=3$ instead of $n=2$? There would be 27 possible samples of $n=3$ from a population of 3 scores. (Try it out, figure out all the possible samples of $n=3$ from the population of 1,2,3. Then compute the mean for each

sample.) Notice how the sample means are closer to μ . For this sampling distribution, seen below, the standard error would be .47.



What would be the probability of obtaining a sample mean of 3 from this sampling distribution of the mean with $n=3$? You should realize that it will be increasingly unlikely to get a mean of 3 with increasing sample size. In this case, the probability of getting a sample mean of 3 would be .04 (1 in 27). There is only one way to get a sample mean of 3...randomly sample 3 on all three selections from the population. Much more often, you will get a sample mean that approaches μ . In the sampling distribution of means, the most typical score (mode) will always be μ .

The discussion so far should clarify one important fact about distributions — the population is very different from the sampling distribution of means. Both distributions have the same mean, but that’s where the similarity ends. For example, the standard deviation of the sampling distribution (called the standard error) will almost always be smaller than the population standard deviation. So, you need to know the nature of the distribution you’re examining.

To figure out the distribution, ask yourself, “What kind of score am I working with?” If your score of interest is a raw score (a person’s score on an IQ test, the number of seconds for a rat to complete a maze, etc.) then you are dealing with a population, which is comprised entirely of raw scores. So, you should be using a z-score formula for a raw score. If your score of interest is a sample mean (the mean score of some group on an IQ test, the mean number of seconds for a group of rats to complete a maze, etc.) then you are dealing with a sampling distribution of means, which is comprised entirely of sample means. So, you should use the z-score formula for a sample mean.

For raw score: $z = \frac{X - \mu}{\sigma}$

For sample mean: $z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$