

Standard Scores, z-Scores, & Probability

Standard Scores. When you want to be able to compare scores from different distributions, or when you want to compute probabilities of particular scores, you have to translate the **scores** into **standard scores**. In general, to translate a raw score into a standard score, you would use the following formula:

$$\text{Standard Score} = \frac{\text{Score you want to standardize} - \text{Mean of distribution}}{\text{Standard deviation of distribution}}$$

z-scores. A particular type of standard score is a **z-score**. When you have **raw scores** from a sample or a population, you can compute a z-score for each raw score. For a sample, the formula for a z-score would be:

$$z = \frac{X - \bar{X}}{s}$$

For a population, the formula would be:

$$z = \frac{X - \mu}{\sigma}$$

In the very unlikely event that you *knew* μ and σ , you could also compute a z-score for individual means found in a sampling distribution of the mean.

The shape of the z-score distribution will be exactly the same as the distribution from which it arose. That is, if the original distribution is positively skewed, the z-score distribution will also be positively skewed. However, *all* z-score distributions will have a mean of 0 and a standard deviation of 1. You should think of the z-score as representing the raw score in terms of standard deviation units from the mean. Thus, a z-score of +1 means that the raw score is 1 standard deviation above the mean.

If the population is **normal**, the z-score distribution will also be normal. When the z-score distribution is normal, you can compute probabilities of obtaining particular z-scores. You do this by using the unit-normal table found in Table B.1. Remember that the normal distribution is symmetrical, so the table only shows the top half of the distribution. To deal with negative z-scores, simply imagine a mirror-image of the table, where the shaded segment B goes from the mean *down* to the z-score, and the shaded segment C goes from the z-score *down* to the very end of the distribution.

Mastery Assessment

1. Suppose that you are dealing with a normal population of IQ scores, with $\mu = 100$ and $\sigma = 15$.

a. Compute the z-score for the following IQ scores, and tell me what percentage of the population would have higher IQ's

130

145

85

70

b. Compute the raw IQ scores for the following z-scores, and tell me what percentage of the population would have lower IQ's.

-1.64

-0.50

0.00

1.00

1.96

2. Suppose you are dealing with a normal population of 100 scores with $\mu = 500$ and $\sigma = 100$.

a. How many scores would you expect to be above 600?

b. How many scores would you expect to be below 450?

c. How many scores would be between 300 and 550?

d. How many scores would be between 100 and 350?

e. How many scores would be between 500 and 1000?

f. What would be the scores representing the upper 25% of the distribution?

g. What scores would represent the middle 20% of the distribution?

h. If someone got a score of 630, how well did they do?