

Laboratory for Homogeneity of Variance, Treatment Effect, and Power

Purpose: To work on computation of the Brown-Forsythe test and to help clarify the concepts of treatment effect and power, in part through the use of SPSS and G•Power.

1. Homogeneity of Variance & Brown-Forsythe Test

The homogeneity of variance assumption seems to be fairly important, especially because it might have implications for the construction of F-ratios for comparisons. Thus, we should be inclined to test $H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \dots$. Although there are a number of possible tests for the homogeneity of variance assumption, K&W propose that we use the Brown-Forsythe test. Computing the actual B-F test in SPSS is a bit cumbersome, but fairly simple. It has the added advantage of familiarizing you with using SPSS to construct new variables that result from transformations of existing variables. Your notes from Ch. 7 use K&W51 to provide a template for computing the B-F test in SPSS. To be sure that the procedure makes sense to you, compute the B-F test for the following data set (available on the computer as BF.lab.sav).

	Group 1	Group 2	Group 3
	1	1	7
	2	9	8
	3	4	9
	4	13	8
	3	8	7
	5	2	8
	2	9	9
	3	1	8
	4	0	7
	3	20	8
Variance			
Median			
Mean			

For your Brown-Forsythe test:

<i>F</i> -ratio	<i>p</i> value

What conclusion would you reach about the homogeneity of variance of this data set?

Next compute an ANOVA on the data above.

<i>F</i> -ratio	<i>p</i> value

How would your conclusions regarding homogeneity of variance influence your interpretation of the ANOVA?

Next re-compute your ANOVA on the original data, but first choose the option of including the Brown-Forsythe test and a test for homogeneity of variance. Can you see how SPSS approaches using the Brown-Forsythe test (correcting *p*)? Does the Levene test tell you the same thing as the B-F test?

To assess the impact of heterogeneity of variance on comparisons, compute the following comparisons {1, -1, 0}, {1, 0, -1} using the formulas (below). (Are these two comparisons orthogonal?)

Comparison Using t	Comparison Using F
$t = \frac{\hat{\psi}}{s_{\hat{\psi}}}$ $\hat{\psi} = \sum c_j \bar{Y}_j$ $s_{\hat{\psi}} = \sqrt{\sum c_j^2 s_{M_j}^2} = \sqrt{\sum c_j^2 s_j^2 / n_j}$	$F = \frac{MS_{Comparison}}{MS_{Error}}$ $MS_{Comparison} = \frac{n\hat{\psi}^2}{\sum c_j^2}$ $\hat{\psi} = \sum c_j \bar{Y}_j$ $MS_{Error} = \frac{\sum (c_j^2)(s_j^2)}{\sum c_j^2}$

Comparisons Using t

	{1, -1, 0}	{1, 0, -1}
Ψ_j		
$s_{\hat{\psi}}$		
t		

Comparisons Using F

	{1, -1, 0}	{1, 0, -1}
$MS_{Comparison}$		
Pooled MS_{Error}		
F		
$MS_{Comparison}$		
Separate MS_{Error}		
F		

Of course, you still don't know if a statistic is significant without a critical value. To determine a critical value (either t or F), you'll need df_{Error} . Doing so requires the use of that magilla formula:

$$df_{Error} = \frac{s_{\hat{\psi}}^4}{\sum \frac{c_j^4 s_{M_j}^4}{n_j - 1}}$$

Finally, compute these contrasts in SPSS and note that you get an output that illustrates a statistic assuming homogeneity of variance and another assuming heterogeneity of variance. Compare the fractional df_{Error} you computed with the df_{Error} in SPSS.

2. Next, let's focus on the factors that affect the treatment effect.

a. Use SPSS to compute an ANOVA on these data (IV with DV1 in Power.Effect.Lab.sav):

G1	G2
1	2
2	3
3	4
4	5
5	6
1	2
2	3
3	4
4	5
5	6

As you can see, the only difference between the two groups is that the scores in G2 ($M = 4$) have a constant of +1 added to the scores of G1 ($M = 3$). That's largely what we think of as a treatment effect (means differ but variances are the same). First, estimate the standardized difference between the means as an index of treatment effect size. Because sample size is the same for the two groups, you can use the square root of the average variance as your standard deviation (the middle formula).

$$d_{1,2} = \frac{\bar{Y}_1 - \bar{Y}_2}{s_{1,2}} \quad s_{1,2} = \sqrt{\frac{s_1^2 + s_2^2}{2}} \quad s_{1,2} = \sqrt{\frac{SS_1 + SS_2}{df_1 + df_2}}$$

$d_{1,2} =$

Estimate the treatment effect (ω^2) for this set of data. At the same time, record the F and MS_{Error} from the source table.

$\hat{\omega}^2$	F	MS_{Error}

b. Now, analyze the following data set (IV with DV2 in Power.Effect.Lab.sav):

G1	G2
1	3
2	4
3	5
4	6
5	7
1	3
2	4
3	5
4	6
5	7

As you can see, this analysis differs only in that the scores in G2 have a constant of +2 added to the scores of G1. Thus, you should understand that this is a larger treatment effect and your estimate of effect size should reflect that fact. Estimate the treatment effect (using d and $\hat{\omega}^2$) to see if it's larger.

$$d_{1,2} =$$

At the same time, record the F and MS_{Error} from the source table.

$\hat{\omega}^2$	F	MS_{Error}

c. Now analyze the following data set (IV with DV3 in file Power.Effect.Lab.sav):

G1	G2
2.600	3.600
2.800	3.800
3.000	4.000
3.200	4.200
3.400	4.400
2.600	3.600
2.800	3.800
3.000	4.000
3.200	4.200
3.400	4.400

In comparison to the analysis that you conducted in 2a, these data have identical means (3 for G1 and 4 for G2). However, the data in each group are less variable. Even though the treatment effect is produced by adding +1 to each data point in G1, as was done in exercise 2a, will your estimate of ω^2 remain the same? Estimate the treatment effect (using d and $\hat{\omega}^2$) to see if it's the same.

$$d_{1,2} =$$

At the same time, record the F and MS_{Error} from the source table.

$\hat{\omega}^2$	F	MS_{Error}

Can you articulate why the same additive constant would yield such different estimates of ω^2 ? The comparison of 2a and 2c is similar to the comparison from Lab 1 (Exercise 4).

3. As you know, one important contribution of the notion of treatment effect is that it is independent of sample size (n). To see the impact of n on $\hat{\omega}^2$, I've created a new file that has

all the same data found in the problems in exercise 1, but duplicated (so the $n = 20$ instead of $n = 10$). All of these data are in the file Power.Effect.Lab2.sav. Enter the information from the three analyses below. How are these values affected by the increase in n ? Does $\hat{\omega}^2$ remain fairly stable?

Exercise	$\hat{\omega}^2$	F	MS_{Error}
1a. with $n = 20$			
1b. with $n = 20$			
1c. with $n = 20$			

4. Now, let's shift our focus to the concept of power. As you know, we can use the concept of power in a predictive, a priori, fashion — largely by assuming a medium effect size and estimating the sample size needed to achieve power of .80. Using ω^2 , where a medium effect size would be .06, you would need $n = 44$ for an experiment with 4 levels of the IV (as in K&W51). Let's use G•Power to see what estimate of n we would get. Start the program and you'll see a window in which the left side allows you to select a-priori computations. Select the F-Test (ANOVA), and you'll note that the program uses f , in which a medium effect size is .25 (which would be the equivalent of $\omega^2 = .06$). Then adjust the values on the right side of the screen so that power is 0.8000 and the number of Groups is 4 (you shouldn't need to adjust Alpha or Effect Size). Then, click on the Calculate button and you'll get your estimate of the total sample size needed (N.B. this is not n , but is $(a)(n)$ or N). How does this estimate of necessary sample size compare with your estimate based on ω^2 ?

5. Now, with a little coaxing, you can also get G•Power to compute power estimates for completed analyses. Here's the procedure. First, click on the Post-Hoc button on the left (and it should be set to ANOVA). Let's start with the K&W51 data set. How much power would you expect to find in a study that produced $F = 7.34$ with $n = 4$? Ahhh, the benefits of fabricating data!

On the right, Alpha is .05, and the total sample size is 16. But you're going to need G•Power to calculate f for you, so click on the Calc " f " button. Adjust the number of groups to 4 and set sigma (estimate of population standard deviation) to the square root of MS_{Error} (12.27 here). Then fill in the sample means (26.5, 37.75, 57.5, and 62.5) and sample n 's (all $n = 4$). (Click on the box, enter the correct value, click on the check box, which will place the values below.) Then, click on "Calc and Copy" and you'll be back to the main window. Click on Calculate and you'll have G•Power's estimate of power.