MA 108 Review Sheet 1 Answers

1. Explain as precisely as you can the concept of function, using the terms "rule", "domain", and "range" in your explanation.

A function is a rule (or a process) that takes inputs to outputs. In this course, the inputs and outputs are real numbers. The set of legal (or "reasonable") inputs for a function is called the domain of the function, and the set of outputs is called the range.

2. Sketch a possible graph of the function described by the following statement: “The average daily temperature in Saratoga Springs last November fluctuated quite a bit but was always above 20°F and less than 65°F.

November has thirty days, so there are 30 data points (one average daily temperature per day). Each one of these values lies between 20 and 65. We have a function in which the input is the day (between 1 and 30) and the output is the average temperature for that day. A possible graph would look like this:

![Graph of average daily temperature in Saratoga Springs]

3. Find an equation for the line that passes through the point (3,8) and has slope –3/5. Find the x- and y-intercepts of this line.

The line has point-slope equation of the form $y - y_0 = m(x - x_0)$, or $y - 8 = \frac{-3}{5} (x - 3)$, or $y = \frac{-3}{5} x + \frac{49}{5}$. The y-intercept is therefore $49/5 = 9.8$, and the x-intercept can be found by solving the equation $0 = \frac{-3}{5} x + \frac{49}{5}$, or $\frac{3}{5} x = \frac{49}{5}$, or $x = 49/3$. 
4. Give an example of an even function and an example of an odd function.

\[ f(x) = x^2 \text{ is even, since } f(-x) = f(x). \]
\[ g(x) = x^3 \text{ is odd, since } f(-x) = -f(x). \]

5. Solve the following inequalities, and express the solution sets in interval notation:

- \[ 1 + 5x > 5 - 3x \implies 8x > 4 \implies x > 1/2. \] In interval notation, \((1/2, +\infty)\).

- \[ x^2 < x + 6 \implies x^2 - x - 6 < 0 \implies (x - 3)(x + 2) < 0. \]

For the product to be negative, the two factors must differ in sign. One sees fairly easily that the two factors differ in sign on the interval \((-2,3)\).

- \[ 1 < |x - 5| < 3 \]

Recall that the absolute value of a quantity is the distance that quantity lies from the origin. So, the inequality tells us that \(x - 5\) lies greater than one unit from the origin, and less than three units from the origin. Therefore, either \(1 \leq x - 5 \leq 3\) or \(-3 \leq x - 5 \leq -1\). By adding 5 to each term in the first of these double inequalities, we find that \(6 \leq x \leq 8\) is in the solution set. Similarly, the second double inequality tells us that \(2 \leq x \leq 4\) is in the solution set. So, using interval notation, the solution set is \([2, 4] \cup [6, 8]\).

6. Solve the following equations:

- \[ |x - 1| = 2. \]

This equation says that the distance between 1 and \(x\) is equal to 2. Therefore, either \(x = -1\) or \(x = 3\).
3. \[ |x + 3| = |2x + 1|. \]

If \(|A| = |B|\), there are two possibilities: either \(A = B\) or \(A = -B\). Why? Because \(|A| = |B|\) means that \(A\) and \(B\) lie at equal distances from the origin on the number line, so they either lie on the same side of 0 (in which case \(A = B\)) or on opposite sides (in which case \(A = -B\)). Therefore, we have two equations to solve. The first is \(x + 3 = 2x + 1\), which yields \(x = 2\). The second is \(x + 3 = -(2x + 1)\), or \(x + 3 = -2x - 1\), or \(3x = -4\), which yields \(x = -4/3\). Therefore, the solution set is \(\{2, -4/3\}\).

7. Find an equation of the line that is perpendicular to the line \(y = 2x - 1\) and passes through the point \((-2, 5)\). Sketch both lines and find their point of intersection.

The perpendicular line has slope \(-1/2\) and equation \(y - 5 = (-1/2)(x + 2)\), or \(y = (-1/2)x + 4\). The two lines meet at the point \(x\) at which the corresponding \(y\)-coordinates are equal, so we solve the equation \(2x - 1 = (-1/2)x + 4\). Multiplying both sides by 2, we get \(4x - 2 = -x + 8\), or \(5x = 10\), or \(x = 2\). Plugging back this value into either equation gives us the \(y\)-coordinate 3. So the lines meet at the point \((2, 3)\).

8. Find the distance between the points \(P = (3, 8)\) and \(Q = (-2, -1)\), and the equation of the circle with center \(P\) that passes through \(Q\).

By the Law of Pythagoras, the distance between the points \((a, b)\) and \((c, d)\) is \(\sqrt{(a - c)^2 + (d - b)^2}\). Therefore, the distance between \(P\) and \(Q\) is...
The standard form of the equation of the circle of radius $r$ that passes through $(a, b)$ is $(x - a)^2 + (y - b)^2 = r^2$. Therefore, the standard form of the circle with center $P$ and passing through $Q$ is: $(x - 3)^2 + (y - 8)^2 = 106.$

9. Simplify the following expressions. Make sure your answer is in lowest terms.

\[
\begin{align*}
\frac{x}{x-1} - \left(\frac{6x+7}{x^2-x}\right) &= \frac{x^2}{x^2-x} - \frac{6x+7}{x^2-x} = \frac{x^2+6x-7}{x(x-1)} = \frac{x+7}{x} \\
\frac{6x^2-7x-20}{3x^2-5x-12} &= \frac{(3x+4)(2x-5)}{(3x+4)(x-3)} = \frac{2x-5}{x-3} \\
\left(\frac{2ab}{a^2-b^2}\right)\left(\frac{a+b}{a^2+b^2}\right) &= \left(\frac{2ab}{(a+b)(a-b)}\right)\left(\frac{a+b}{a^2+b^2}\right) = \frac{2}{(a-b)a b^2}
\end{align*}
\]

10. If $f$ is the function with the graph shown, sketch graphs of the following transformed versions of $f$:

\[
\text{In[11]}:= \ f[x_\_] := \text{If}[x < 2, \ (1/2) \ x, \ 1]
\]
- \( g(x) = 2f(x) + 1 \)

\[\text{In}[13]= \]
\[
\text{Plot}\left[\{f[x], 2f[x] + 1\}, \{x, -4, 6\}, \text{PlotLabel} \to "y = f(x) \text{ and } y = g(x)"\right], \\
\text{AspectRatio} \to \text{Automatic}, \text{PlotRange} \to [-5, 5]\]

\[\text{Out}[13]= \]

\[\text{y = f(x) and y = g(x)} \]

- \( h(x) = -f(x - 2) \)

\[\text{In}[114]= \]
\[
\text{Plot}\left[\{f[x], -f[x - 2]\}, \{x, -4, 6\}, \text{PlotLabel} \to "y = f(x) \text{ and } y = h(x)"\right], \\
\text{AspectRatio} \to \text{Automatic}, \text{PlotRange} \to [-5, 5]\]

\[\text{Out}[114]= \]

\[\text{y = f(x) and y = h(x)} \]
11. Match each function to its family.

- \( \frac{3x+2}{x^2 - 5} \) This is a rational function (also algebraic).

- \( \tan(3x - 1) \) This is a transcendental function.

- \( 2x - 7 \) This is a linear function (also polynomial, etc.).

- \( x^{2/3} \) This is a power function (also algebraic).

- \( x^3 - 8x^2 + 3x - 9 \) This is a polynomial function (also rational, etc.).

- \( \sqrt{\frac{8x-3}{7x^{2/3} - 8}} \) This is an algebraic function.