1. Express each of the following so that each variable appears at most once and with a non-negative exponent:

\[
\frac{a^5 b^{-3}}{a^2 b^2 c^{-2}} = \frac{a^3 c^2}{b^7}
\]

\[
\frac{1}{b^{-1}} \cdot \frac{a^2 b^6}{b^{-3} c^4} = \frac{1}{a b^2} \cdot \frac{a^3 b^8}{c^4} = \frac{a^5 b^6}{c^4}
\]

\[
(a b^4 c^{-3})^9 = a^9 b^{36} c^{-27} = \frac{a^9 b^{26}}{c^{27}}
\]

2. If a pulley with radius 5 inches is pulled through a rotation of \(3\pi/4\) radians, about how far (in inches) does the pulley lift the bucket?

A point on the edge of the pulley travels a distance of \(r \theta = 5 \cdot (3 \pi/4) = 15 \pi/4\) inches. This is also the length of rope pulled in, and so equals the number of inches that the bucket is raised. Note that \(15 \pi/4\) is approximately equal to 11.781

3. Suppose Jorge is standing 80 feet from the base of a tree. If his angle of sight to the top of the tree is 30° and he stands 5 feet tall, about how tall is the tree?

Let \(x\) be the opposite side of the triangle. Then the height of the tree is \(x + 5\), and \(x/80 = \tan(30°)\), so \(x = 80 \cdot \tan(\pi/6)\). Therefore, the height of the tree in feet is 51.188
4. Convert each of the following angles to degree measure. Also draw these angles in standard position, and find the sine, cosine, and tangent of each:

- \[ \frac{5\pi}{4} \]

\[ \frac{5\pi}{4} \text{ radians is equal to the following number of degrees:} \]

\[ \left\lfloor \frac{5\pi \cdot 180}{4 \cdot \pi} \right\rfloor = 225. \]

Here is a picture of the angle in standard position; the selected point is \((-1, -1)\), which lies at distance \( r = \sqrt{2} \) from the origin:

Therefore, we have \( \sin\left(\frac{5\pi}{4}\right) = \frac{y}{r} = -1/\sqrt{2} \), \( \cos\left(\frac{5\pi}{4}\right) = \frac{x}{r} = -1/\sqrt{2} \), and \( \tan\left(\frac{5\pi}{4}\right) = \frac{y}{x} = -1/-1 = 1. \)

- \[ -\frac{13\pi}{6} \]

\[ -\frac{13\pi}{6} \text{ radians is equal to the following number of degrees:} \]

\[ \left\lfloor \frac{-13\pi \cdot 180}{6 \cdot \pi} \right\rfloor = -390. \]

Here is a picture of the angle in standard position; the selected point is \((\sqrt{3}, -1)\), which lies at distance \( r = 2 \) from the origin:
Therefore, we have $\sin(-13\pi/6) = y/r = -1/2$, $\cos(-13\pi/6) = x/r = \sqrt{3}/2$, and $\tan(-13\pi/6) = y/x = -1/\sqrt{3}$.

-5\pi

-5\pi radians is equal to the following number of degrees:

$\frac{-5\pi \cdot 180}{\pi} = -900$.

Here is a picture of the angle in standard position; the selected point is (-1, 0), which lies at distance $r = 1$ from the origin:

Therefore, we have $\sin(-5\pi) = y/r = 0/1 = 0$, $\cos(-5\pi) = x/r = -1/1 = -1$, and $\tan(-5\pi) = y/x = 0/-1 = 0$. 
5. Express as a single logarithm: \( \log_5(7) + \log_5(3x) - \log_5(2y) \).

\[
\log_5 \left( \frac{7 \cdot 3x}{2y} \right)
\]

6. Complete the following trigonometric identities:

- \( \cos(2x) = 1 - 2\sin^2(x) \)
- \( \sin(x + \pi) = -\sin(x) \) -- i.e., the sine function is even.
- \( \sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B) \)
- \( \tan^2(x) + 1 = \sec^2(x) \)

7. Let \( f(x) = \sqrt{3x + 7} \). Find the domain and range of \( f \), a formula for the inverse function \( f^{-1}(x) \), and the domain and range of \( f^{-1} \).

The domain of \( f \) consists of all \( x \) such that \( 3x + 7 \geq 0 \), or \( x \geq -7/3 \), or \( [-7/3, \infty) \). The range is \( [0, \infty) \). One sees that \( f \) is one-to-one on its entire domain, and so is invertible (see graph). The domain of the inverse function \( f^{-1} \) is \( [0, \infty) \) and the range is \( [-7/3, \infty) \) -- the inverse function’s domain is the range of the original function, and its range is the domain of the original function.

To get a formula for \( f^{-1} \), we start with \( y = \sqrt{3x + 7} \), and we solve for \( y \) as a function of \( x \) -- think of \( y \) as the input, and \( x \) as the output. We get \( y^2 = 3x + 7 \), or \( 3x = y^2 - 7 \), or \( x = f^{-1}(y) = \frac{y^2 - 7}{3} \). So, using \( x \) as the input to \( f^{-1} \), we have that \( f^{-1}(x) = \frac{x^2 - 7}{3} \).
8. Find the derivative of each of the following functions. You need not simplify your answers algebraically.

a. \((7x^5+4x^2)' = (7x^5 + 4x^2)' = 35x^4 - 8x^3\)

b. \((\sin(\ln(3x)))' = \cos(\ln(3x)) \cdot (\ln(3x))' = \cos(\ln(3x)) \cdot (1/(3x)) \cdot 3 = \frac{\cos(\ln(3x))}{x}\)

c. \((3x^2 + 5)^3 (6x) = 24x(3x^2 + 5)^3\)

d. \((\cos(x)/x)' = \frac{(\cos(x)') \cdot x - \cos(x) \cdot x'}{x^2} = \frac{-\sin(x) \cdot x - \cos(x) \cdot 1}{x^2}\)

e. \((3^{2x+5})' = 3^{2x+5} \cdot \ln(3) \cdot (2x+5)' = 2 \ln(3) \cdot 3^{2x+5}\)

f. \((x^2 \cdot \arctan(2x))' = (2x) \arctan(2x) + x^2 \left( \frac{1}{1+(2x)^2} \cdot (2) \right)' = 2x \arctan(2x) + \frac{2x}{1+4x^2}\)

9. Use implicit differentiation to find an expression for \(y'\), where \(y\) is the function of \(x\) defined implicitly by the relation \(x^2 y + y^2 = 2\). Then find the equation of the line tangent to the graph of the relation at the point \((1, 1)\).

Start with the relation \(x^2 y + y^2 = 2\). Think \(y = f(x)\) and differentiate both sides with respect to \(x\): \(2x \ y + x^2 \ y' + 2y \ y' = 0\), so \(y' = \frac{-2xy}{x^2 + 2y}\). At the point \((1,1)\), \(y' = -2/3\), so the equation of the tangent line at \((1,1)\) is \(y - 1 = (-2/3)(x - 1)\).

10. Find the exponential function \(f(x) = A \cdot b^x\) such that \(f(0) = 7\) and \(f(2) = 63\). Sketch its graph, and find its derivative at \(x = 1\).

Since \(f(0) = A \cdot b^0 = A \cdot 1 = A\), we have that \(A = 7\) (the initial value of the function). Therefore, \(f(2) = 7 \cdot b^2 = 63\), which implies that \(b^2 = 9 \Rightarrow b = 3\). So the function is \(f(x) = 7 \cdot 3^x\). The derivative is \(f'(x) = 7 \cdot 3^x \cdot \ln(3)\); therefore, \(f'(1)\) is equal to

\[
\text{N[21 Log[3]]}
\]

23.0709

Here's the graph, along with the tangent line at \(x = 1\).
11. Let the function $g$ be defined by $g(x) = A \log_7(x) + B$, and suppose that $g(1) = 3$ and $g(1/7) = -2$. Determine $A$ and $B$, and find $g'(2)$.

If $g(1) = 3$, then $A \log_7(1) + B = A \cdot 0 + B = 3$, so $B = 3$. Therefore $g(x) = A \log_7(x) + 3$, so $g(1/7) = A (-1) + 3 = -2$, which implies that $A = 5$. Therefore, $g(x) = 5 \log_7(x) + 3$, so $g'(x) = \frac{5}{x \ln(7)}$, so $g'(2) = \frac{5}{2 \ln(7)} = \approx 1.28475$.

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In[3] := g[x_] := 5 Log[7, x] + 3

In[6] := Plot[{g[x], g[2] + (1.28475) (x - 2)}, {x, .1, 5}]
Out[6] = 
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