The second midterm exam will cover everything we have done from the first midterm exam through the topic of L’Hospital’s Rule (section 4.4 of the text).

**Topics:**

1. Exponential growth & decay.
2. Related rate problems.
3. Linear approximation and differentials.
4. Maximum and minimum values of a function (both global and local); critical points; extreme value theorem.
5. Mean Value Theorem (MVT).
6. Use of the first and second derivatives in curve sketching; detecting critical points, intervals of increasing and decreasing behavior, concavity, and points of inflection.
7. L’Hospital’s rule for limits of the form “0/0” and “∞/∞”; other indeterminate forms.

**Sample Problems:**

1. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420. (a) Find an expression for the number of cells in the colony after \( t \) hours. (b) Find the number of bacteria after 3 hours. (c) Find the rate of growth at \( t = 3 \) hours. (d) When will the population reach 10,000 cells?
2. If \( y = x^3 + 2x \) and \( dx/dt = 5 \), find \( dy/dt \) when \( x = 2 \).
3. A spherical balloon is being inflated so that its volume increases at a rate of 2 ft\(^3\)/min. How fast is the radius increasing when the diameter of the balloon is 4 ft across?
4. Find the critical points of the following functions: \( g(t) = |3t - 4| \) and \( h(x) = \frac{x-1}{x^2 + 4} \).
5. Find the global maximum and global minimum values of the function \( f(x) = x - \ln(x) \) on the interval \([1/2, 2]\).
6. Repeat the preceding problem for the function \( g(x) = x\sqrt{4-x^2} \) on the interval \([-1, 2]\).
7. Find the point that the Mean Value Theorem guarantees will exist for the function \( f(x) = \sqrt{x} \) on the interval \([1, 9]\). Draw a picture illustrating the MVT in this case.
8. The graph of the second derivative \( f'' \) of the function \( f \) is shown. What are the \( x \)-coordinates of the points of inflection of \( f \)? On what intervals is \( f \) concave up and concave down? Explain your answers.
9. Let \( f(x) = 2 + 2x^2 - x^4 \). Find (a) the interval(s) on which \( f \) is increasing and decreasing; (b) the local maximum and minimum value(s) of \( f \), and (c) the intervals of concavity and the point(s) of infection.

10. The graph of the first derivative \( g' \) of the function \( g \) is shown. Identify the local extreme points of \( g \), and make a rough sketch of the graph of \( g \), assuming that \( g(0) = -2 \).

11. Evaluate the following limits: (a) \( \lim_{x \to 0} \frac{\sin(4x)}{\tan(5x)} \) (b) \( \lim_{x \to 0} \frac{1 - \cos(x)}{x^2} \) (c) \( \lim_{x \to 0} x^{1/x} \)