Problem 1.

\[
\int \frac{\sin \left( \sqrt{x} + 1 \right)}{\sqrt{x}} \, dx
\]

Set \( u = \sqrt{x} + 1 \), \( du = \frac{1}{2\sqrt{x}} \, dx \).

\[ -2 \cos \left[ 1 + \sqrt{x} \right] \]

\[
\int \left( x e^{2x} \right) \, dx
\]

Use integration by parts with \( u = x \), \( dv = e^{2x} \, dx \).

\[ e^{2x} \left( -\frac{1}{4} + \frac{x}{2} \right) \]

\[
\int \frac{x}{x^2 + 3x + 2} \, dx
\]

Use integration by partial fractions.

\[ -\log(1 + x) + 2 \log(2 + x) \]

\[
\int_{0}^{\pi/2} \frac{\cos t}{\sqrt{3 \sin t + 1}} \, dt
\]

Use substitution \( u = 3 \sin t + 1 \), and update the limits of integration.

\[
\frac{2}{3}
\]

\[
\int \frac{1}{\sqrt{4 + x^2}} \, dx
\]
Use the trig substitution \( x = 2 \tan[t] \), \( dx = 2 \sec^2[t]dt \), to obtain \( \log\left[ \frac{\sqrt{4+x^2}}{2} + \frac{x}{2} \right] \), which is the same as ArcSinh[x/2] as reported by Mathematica.

\[
\int \cos[x]^3 \, dx
\]

Split off one copy of \( \cos[x] \) to go with the \( dx \), rewrite the \( \cos^2[x] \) as \( 1 - \sin^2[x] \), and set \( u = \sin[x] \), \( du = \cos[x] \, dx \).

\[
\sin[x] - \sin[x]^3/3 + C
\]

\[
\int_1^\infty e^{-2x} \, dx = \lim_{t \to \infty} \int_1^t e^{-2x} \, dx = \lim_{t \to \infty} \left( \frac{1}{2} \left( \frac{1}{e^2 - 1} \right) \right) = \frac{1}{2} e^2
\]

\[
\sum_{n=2}^{\infty} \left( \frac{2}{5} \right)^n
\]

A converging geometric series with \( a = 4/25 \) and \( r = 2/5 \).

\[
\frac{4}{15}
\]

Problem 2.

\[
\int_1^\infty \frac{x}{x^3+10} \, dx
\]

The integrand satisfies the inequality \( \frac{x}{x^3+10} < \frac{x}{x^2} = \frac{1}{x} \) for \( x \geq 1 \), so we know the improper integral converges by comparison to the known converging integral \( \int_1^\infty \frac{1}{x^2} \, dx \).

We can make our tail less than a given amount by making the corresponding tail of the "test function" that small. So, to find \( d \) as requested, we compute

\[
\int_d^\infty \frac{1}{x^2} \, dx = \frac{1}{d}
\]

and solve the inequality \( \frac{1}{d} < .001 \), to obtain \( d > 1000 \). Any value of \( d \) larger than 1000 will do.

Let's test this:
\[ \ln[2] = N\left[ \int_{1001}^{\infty} \left( \frac{x}{x^3 + 10} \right) \, dx, 10 \right] \]

\[ \text{Out}[2] = 0.0009990009965 \]

We see that this "tail" is less than .001.

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**Problem 3.**

The slices of the x-axis volume are disks of radius \( e^x \). The slices of the y-axis volume are disks of radius 1 between \( y = 0 \) and \( y = 1 \), and are washers of inner radius \( \ln[y] \) and outer radius 1 for \( 1 \leq y \leq e \). (The y-axis volume is easier to compute using the "shell method.")

\[
\text{xAxisVolume} = \pi \int_{0}^{1} e^{2x} \, dx \\
\left( - \frac{1}{2} + \frac{e^2}{2} \right) \pi \\
\text{yAxisVolume} = \pi + \pi \int_{1}^{e} (1^2 - \ln[y]^2) \, dy \\
2 \pi
\]

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**Problem 4.**

If \( f[x] = \frac{3}{x} \), then \( f'[x] = -\frac{3}{x^2} \). The length of the graph of \( f \) on the interval \([1,3]\) is therefore

\[
\int_{1}^{3} \sqrt{1 + f'[x]^2} \, dx = \\
\int_{1}^{3} \sqrt{1 + \frac{9}{x^4}} \, dx 
\]

The length can be estimated by adding up the distances between the points \((1,3), (2, \frac{3}{2})\), and \((3,1)\):

\[
\text{approxArcLength} = N \left[ \sqrt{(2 - 1)^2 + \left( \frac{3}{2} - 3 \right)^2} + \sqrt{(3 - 2)^2 + \left( 1 - \frac{3}{2} \right)^2} \right] \\
2.92081 
\]

The Right Riemann Sum \( R_2 \) for the arc length integral is \((\Delta x = 1, \text{and the right endpoints of the two subintervals are } x = 2 \text{ and } x = 3)\):

\[
N \left[ \frac{(3 - 1)}{2} \left( \sqrt{1 + \frac{9}{(2)^4}} + \sqrt{1 + \frac{9}{3^4}} \right) \right] \\
2.30409 
\]
The Right Riemann Sum gives an underestimate of the arc length (as does the polygonal approximation), since the integrand is decreasing. The exact value of the arc length is

\[
\ln[4]=
\int_{1}^{3} \sqrt{1 + \frac{9}{x^4}} \, dx, 10
\]

Out[4]: 2.955334906

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**Problem 5.**

The problem is to find the number of subintervals needed to estimate the value of the (easy) integral \( \int_{0}^{2} (4 - x^2) \, dx \) to within .001 using the Midpoint method. The error bound for the Midpoint method is given by \( \frac{K_2(b-a)^2}{24n^2} \). Now the second derivative of the integrand is the constant -2, so \( K_2 = 2 \). We must solve the inequality \( \frac{2(2-a)^2}{24n^2} < .001 \), or \( n^2 > \frac{16000}{24} \), or \( n > \sqrt{\frac{16000}{24}} = 25.8199 \). Therefore \( n = 26 \) will do.

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**Problem 6.**

\[1/3, 2/5, 3/7, 4/9, 5/11, \ldots, n/(2n+1), \ldots\]
\[1, -1/2, 1/6, -1/24, 1/120, \ldots, (-1)^{n+1} / n!, \ldots\]

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**Problem 7.**

\{2 - 1/n\} converges monotonically to 2: 1, 3/2, 5/3, 7/4, ...
-1, -2, -3, -4, -5, ... or -1, -4, -9, -16, -25, ... diverge to -\(\infty\)
\[\sum_{k=1}^{\infty} \frac{1}{(k/3)^3} \] is a diverging p-series.
\[\sum_{k=1}^{\infty} \left(-\frac{1}{3}\right)^k \] is a converging alternating geometric series.

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**Problem 8.**

a. diverges by comparison to diverging p-series with \( p = 2/3 \).
b. converges (use the ratio test).
c. diverges (the \( n \)th term tends to \( \infty \) as \( n \to \infty \)).
d. converges by the alternating series test.
Problem 9.

The given power series is geometric, with common ratio given by \( \frac{r}{2} \). The series will converge for all x such that \( |\frac{r}{2}| < 1 \), that is, \(|x-2| < 2 \). In other words, the series converges for all x that lie less than 2 units from the center 2, which yields the open interval \((0, 4)\).

Problem 10.

The slope \( y' \) is twice the y-coordinate, so the curve is a solution of the DE \( y' = 2y \), which has solutions of the form \( y = A \cdot e^{2 \cdot t} \). If \( y = 5 \) when \( x = 0 \), the constant A must be 5. If the slope is twice the x-coordinate, then we have the DE \( y' = 2x \), or \( y = x^2 + C \) with \( C = 5 \) to satisfy the initial condition.

Problem 11.

The number of bacteria at time \( t \) is given by \( P(t) = P_0 \cdot e^{kt} \), where \( t \) is in hours. We are given that \( P(2) = 600 \) and \( P(8) = 75000 \).

Therefore \( \frac{P(8)}{P(2)} = \frac{P_0 \cdot e^{8k}}{P_0 \cdot e^{2k}} = \frac{75000}{600} = 125 \Rightarrow e^{6k} = 125 \Rightarrow k = \frac{\ln(125)}{6} = .804719 \). Solving for \( P_0 \) in \( P(2) = 600 = P_0 \cdot e^{0.804719 \cdot 2} \Rightarrow P_0 = \frac{600}{e^{1.619438}} = 120 \). Therefore, \( P(t) = 120 \cdot e^{0.804719 \cdot t} \). The population at 5 hours is \( P(5) = 6708 \). The growth rate at 5 hours is \( P'(t) = 120k \cdot e^{kt} = 5398 \) bacteria/hour. To find when the population will reach 200,000 bacteria, we solve the equation \( P(t) = 200000 \) for \( t \). The answer is: \( t = 9.21885 \) hours.

Problem 12.

The slope at a point on the parametric curve \( (x(t), y(t)) \) is given by \( \frac{y'(t)}{x'(t)} \). In this case, the slope is \( \frac{\frac{3t^2-12}{2t}}{t} \). The slope is horizontal when the numerator is zero and the denominator isn't 0; these values are \( t = \pm 2 \), corresponding to the points \((6, -16)\) and \((6, 16)\). The slope is vertical when the denominator is 0 and the numerator isn't; the only such point is \( t = 0 \), corresponding to the point \((10, 0)\). Here is (a portion of) the curve:

\[
\text{pcurve} = \text{ParametricPlot}\left[\{10 - t^2, t^3 - 12 t\}, \{t, -3, 3\}\right]
\]

The slope at \( t = -1 \) is \( -\frac{9}{2} \), and the point is \((9, 11)\), so an equation of the line is \( y - 11 = -\frac{9}{2} (x - 9) \).

\[
\text{lplot} = \text{Plot}\left[11 - \frac{9}{2} (x - 9), \{x, 6, 12\}\right]
\]
Problem 13.

To find the radius of convergence of a power series, one uses the ratio test to check where the series converges absolutely. Then, once the radius of convergence \( R \) is known, one needs to check the endpoints \( c-R, c+R \) (where \( c \) is the center of the power series) separately for convergence.

a. center = 1, \( R = 4 \), series converges at \( x = -3 \), diverges at \( x = 5 \), so the interval of convergence is \([-3, 5)\).
b. center = 1, \( R = 1 \), interval of convergence \([0, 2)\).
c. center = -5, \( R = 1 \), interval of convergence \([-6, -4]\).
d. center = 1, \( R = \frac{1}{2} \), interval of convergence \([1/2, 3/2)\).

Here are the details for part a. Given a value of \( x \), we need to test the series for absolute convergence. So we compute

\[
\lim_{k \to \infty} \left( \frac{a_{k+1}}{a_k} \right) = \lim_{k \to \infty} \left( \left| \frac{x-1}{x+1} \cdot \left( \frac{1}{4} \right)^k \right| \right) = \lim_{k \to \infty} \left( \frac{|x-1| \cdot \left( \frac{1}{4} \right)^k}{|x+1|} \right).
\]

We know the series converges absolutely if the preceding limit is < 1; therefore, we see that the series converges absolutely provided that \( |x-1| < 4 \), so the radius of convergence \( R = 4 \). So we have absolute convergence on the interval \((-3, 5)\). One then tests the endpoints separately for convergence. In this case, one sees that we have an alternating harmonic series when \( x = -3 \) and a harmonic series when \( x = 5 \), so we have convergence at -3 and divergence at 5, as reported above.