This review sheet contains a list of topics to be covered on the exam, and some sample problems. In terms of the text, the exam will cover sections 7.3-7.4, 7.7-7.8, and 9.1-9.3. One two-sided 3 x 5 index card of notes may be used while taking the exam.

Exam 2 Topics:

1. **Trigonometric substitutions**: Substitutions for handling expressions containing $\sqrt{x^2 - a^2}$, $\sqrt{x^2 + a^2}$, and $\sqrt{a^2 - x^2}$. Back substitutions.
2. **Integration of rational functions by partial fractions**: Rational functions, long division, recipes for partial fraction decomposition and integration of the resulting partial fractions.
3. **Approximate integration**: Approximate computation of definite integrals using the Trapezoidal, Midpoint, and Simpson approximations. Use of the error bounds to estimate errors and to predict the number of subintervals needed to achieve a desired accuracy. Use of methods with functions given as tables of data.
4. **Improper integrals**: The two cases of improper integrals: infinite intervals and infinite discontinuities. Convergence/divergence of improper integrals; computation of (convergent) improper integrals using limit process; comparison tests for convergence/divergence.

Sample Problems:

1. Evaluate the following integrals:

   \[
   \int_{0}^{\sqrt{3}} \frac{x^3}{\sqrt{16 - x^2}} \, dx \quad \int_{-1}^{1} \frac{x}{\sqrt{x^2 - 7}} \, dx \quad \int_{0}^{1} \left( \sqrt{x^2 + 4} \right) \, dx
   \]

2. Integrate:

   \[
   \int \frac{x^2}{(x-3)(x+2)^2} \, dx
   \]

3. Consider the definite integral \( \int_{1}^{4} \sqrt{x} \, dx \). Does the Midpoint Rule give an overestimate or an underestimate of the exact value? How many subintervals are needed to ensure that the magnitude of the error \( |E_M| \leq 0.00001 \) ? Repeat for the Trapezoidal Rule. (Recall \( |E_M| \leq \frac{K_2 (b-a)^3}{24n^2} \) and \( |E_T| \leq \frac{K_2 (b-a)^3}{12n^2} \), where \( K_2 \) is an upper bound of \( |f'''(x)| \) on \([a,b]\).)
4. Values of the acceleration $a(t)$ of a test car in (ft/sec)/sec $t$ seconds after it begins moving were recorded in the following table. Use Simpson’s Rule to estimate the velocity of the car in ft/sec at time $t = 6$ seconds.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(t)$</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>10</td>
<td>13</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

5. Determine if the improper integrals in the following list converge or diverge. If convergent, find the value of the integral.

\[
\int_{-1}^{0} \frac{1}{x^2} \, dx \quad \int_{0}^{1} \frac{2x}{x^2 + 1} \, dx \quad \int_{0}^{2} \frac{x - 3}{2x - 3} \, dx \quad \int_{0}^{\infty} \frac{x}{(x^2 + 2)^2} \, dx
\]

6. Use Euler’s method with step size 0.2 to estimate $y(0.4)$, where $y = y(x)$ is the solution of the initial value problem $y' = 2xy^2$, $y(0) = 1$. Then repeat with step size 0.1. Find the exact solution of the DE and compare the exact value of $y(0.4)$ with the approximate values.

7. Find the solution of the differential equation $\frac{dy}{dx} = y^2 + 1$ that satisfies the initial condition $y(1) = 0$. 