Chapter 5: z-Scores

The next portion of the course deals with standard scores. In general, you compute a standard score to transform a score from its original units into standard deviation units. Doing so actually allows you to compare apples and oranges (sort of). The formula for standard scores is:

\[
\text{The score you're interested in standardizing} - \text{The mean of the distribution it comes from} \div \text{The standard deviation of the distribution it comes from}
\]

The first standard score you’ll learn is a \( z \)-score for a population.

A \( z \)-score specifies the precise location of each \( X \) value within a distribution. The sign of the \( z \)-score (+ or -) signifies whether the score is above the mean (positive) or below the mean (negative). The numerical value of the \( z \)-score specifies the distance from the mean by counting the number of standard deviation units between \( X \) and \( \mu \).

As you would predict from the general formula for standard scores, the formula to translate a raw score from a population into a \( z \)-score is:

\[
z = \frac{X - \mu}{\sigma}
\]

At this point in the semester, I would hope that you are thinking, “How can we possibly know \( \mu \) and \( \sigma \)? That is, you should recognize that we can rarely know such population parameters, but instead must estimate them. Although that is a legitimate concern, we will first deal with populations for which the parameters are fairly well established. At the same time, you should also note that you could compute a \( z \)-score on sample data, with a formula like:

\[
z = \frac{X - \bar{X}}{s}
\]

If all that you want to do is to compute a standard score for purposes of comparison, it doesn’t much matter if you’re dealing with a sample or a population. (In fact, for some of the examples from the text, it should be clear to you that the authors are really talking about samples, as in the example of test scores, etc.)

The \( z \)-Score Distribution
You should think of a \( z \)-score as a member of a \( z \)-score distribution. The \( z \)-score distribution emerges when you convert every raw score in a distribution into its corresponding \( z \)-score. The \( z \)-score distribution will have certain characteristics:
- the mean of the \( z \)-score distribution will always be 0
- the standard deviation of the \( z \)-score distribution will always be 1
- the shape of the \( z \)-score will always the same as that of the distribution of raw scores
For instance, as seen in this figure from your text, if your distribution had a mean (μ) of 100 and a standard deviation (σ) of 10, and you then computed the z-score for every raw score in the population, you would end up with the distribution on the right. That is, the shape would be the same (symmetrical in this case), but the mean would now be 0 and the standard deviation 1.

- because a z-score is a deviation score, for the z-score distribution \( \sum z = 0 \)

- \( \sum z^2 = SS = N \), which follows from the formula for population variance:

\[
\sigma^2 = 1 = \frac{SS}{N} = \frac{\sum z^2}{N}
\]

**Practical applications of z-scores**

Suppose that you get an exam score of 80 in one class and a score of 70 in another class. On the face of it, I’m sure that you would argue that you did better in the class where you got an 80. However, suppose that the mean and standard deviation in that class were 90 and 10 respectively. Suppose, further, that in the class where you got a 70 the mean and standard deviation were 60 and 5 respectively. If you convert both scores to z-scores, a different picture emerges:

\[
z = \frac{80 - 90}{10} = -1 \\
z = \frac{70 - 60}{5} = +2
\]

It’s now clear that the score of 80 was actually one standard deviation below the class mean. (The exam was obviously too easy! 😊) The score of 70 was actually two standard deviations above the mean, so you did much better than the rest of the class. (Of course, it might still be a C-, so that may be only slight comfort.)

You’ve already seen standard scores like z-scores when you looked at your SAT scores. The mean is recalibrated to 500 and the standard deviation is recalibrated to 100, but otherwise these scores are standard scores like z-scores. IQ scores are quite similar, with a mean of 100 and a standard deviation of 15.