Thursday, 3/20/14
- Return, go over HW #3; HW #4 will be posted later today or tomorrow
- Equivalence relations

Reading:
- [J] 3.4
- [H] 11.2-11.3

Exercises:
- [J] pp. 164-166, #s 1, 2, 9, 10, 15, 16, 38
- [J] p. 180, #s 13-16 (in #16, an "eight-bit string" is a string of eight 0s and 1s, e.g. 01101110 is an eight-bit string)
- [H] p. 185: 1, 5-7, 10-12; p. 188: 3, 5

Equivalence Relations

- **EQUIVALENCE RELATION**: A relation that identifies items that all share a common property, making them equivalent in some sense
- We often use the symbol \(\approx\) to mean "equivalent"

**Example 1**:
- \(S = \mathcal{P}\{1, 2, 3\}\), \(A \approx B\) means \(|A| = |B|\)

**Example 2**:
- \(S = \{1, 2, \ldots, 10\}\), \(p \approx q\) means \(p \equiv q \pmod{4}\)

- Draw their digraphs. What properties do their digraphs have in common?
- Which properties do these examples have: reflexive, symmetric, transitive, antisymmetric?

Definition of Equivalence Relation

- An **equivalence relation** on a set \(A\) is a relation that is
  - Reflexive, and
  - Symmetric, and
  - Transitive.

- The digraph of a relation is composed of disjoint "complete" subgraphs – subgraphs in which every pair of vertices is related.
  - Each subset of equivalent elements is called an **equivalence class**.
Equivalence Relation on S Induces Partition of S

- Equivalence relation:
  - \( S = \{1, 2, \ldots, 10\} \), \( p \sim q \) means \( p = q \) (mod 4)

- Diagram:

- Partition into equivalence classes:
  - \( \{4, 8\}, \{1, 5, 9\}, \{2, 6, 10\}, \{3, 7\} \)

- Do the same for:
  - \( S = \mathcal{P}(\{1, 2, 3\}) \), \( A \sim B \) means \(|A| = |B|\)

A Partition of S induces an equivalence relation on S

- Example:
  - \( S = \{\text{Pierre, Gove, Mark, David, Alice, Rachel, Dan, Leo, Tom, Mike}\} \)

- Partition:
  - \( \{\text{Pierre, Rachel}\}, \{\text{Gove, Mark, Mike}\}, \{\text{David, Alice}\}, \{\text{Leo, Dan, Tom}\} \)

- In general define \( a \sim b \) if and only if \( a \) and \( b \) are in the same subset of the partition.

- Check that this definition always gives a relation that is reflexive, symmetric, transitive.

Equivalence class notation

- We use the notation \([a]\) to denote the name of the equivalence class containing \(a\).
  - E.g., \(\{\text{Gove, Mark, Mike}\}\) can be denoted:
    - \([\text{Gove}]\) or \([\text{Mark}]\) or \([\text{Mike}]\)
  - Any one of the three is a valid name for the set: \([\text{Mike}] = \{\text{Gove, Mark, Mike}\} = [\text{Mark}] = [\text{Gove}]\)

- Observation about equivalence classes for an equivalence relation \(R\):
  - If \((x, y) \in R\), then \([x] = [y]\). Why?
  - If \((x, y) \notin R\), then \([x] \cap [y] = \emptyset\). Why?