Thursday, 1/23/14

- Finish discussing sets
- Propositions and logical operators

Reading:
- [J] 1.2
- Optional: [H] 2.1-2.2

Exercises:
- [J] pp. 30-31: 21-22, 60-61

Homework #1:
- Available on website
- Due by Friday, 1/31/14 at 3 PM
- There is a box outside my office for written submissions, or email submissions are also fine
Propositions

- **SET THEORY:**
  - Provides language for working with collections of objects in a formal way

- **PROPOSITIONAL LOGIC:**
  - Provides the language for formal reasoning in proofs and algorithm analysis

**PROPOSITION:** A sentence that is either true or false
Clicker Question

- **1.** Albany is the capital of New York State.
- **2.** Rafael Nadal is better at tennis than Novak Djokovich.
- **3.** Do you like to ski?
- **4.** There are eight days in a week.
- **5.** Go clean your room!
- **6.** $12 + 34 = 46$.
- **7.** $3x + 5 = 8$.
- **8.** There is an integer solution to the equation $x^2 - 10 = 5$.

On the left are 8 sentences. The propositions are:

- **A.** All of them
- **B.** None of them
- **C.** 1 and 6
- **D.** 1, 4, 6, 8
- **E.** 1, 2, 4, 6, 8
Logical Operators, Conjunction

- **Set operators** create new sets from existing ones.
  - Union, intersection, complement (also power set, Cartesian product)

- **Logical operators** create new propositions from existing propositions.

- **Three important logical operators**: Let $p$ and $q$ be two propositions.
  - **CONJUNCTION (AND)**: “$p$ and $q$”
    - Math notation: $p \land q$; C/Java notation: $p \&\& q$
  - **DISJUNCTION (OR)**: “$p$ or $q$”
    - Math notation: $p \lor q$; C/Java notation: $p \mid\mid q$
  - **NEGATION (NOT)**: “Not $p$”
    - Math notation: $\neg p$; C/Java notation: $! p$
Truth Tables for AND, OR, NOT

- A **truth table** defines the meaning of an operator by showing its value for all possible values of its arguments:

  - **AND:**
    - \[ p \land q \]
    - \[
      \begin{array}{ccc}
        p & q & p \land q \\
        T & T & T \\
        T & F & F \\
        F & T & F \\
        F & F & F \\
      \end{array}
    \]

  - **NOT:**
    - \[ \neg p \]
    - \[
      \begin{array}{cc}
        p & \neg p \\
        T & F \\
        F & T \\
      \end{array}
    \]

  - **OR:**
    - \[ p \lor q \]
    - \[
      \begin{array}{ccc}
        p & q & p \lor q \\
        T & T & T \\
        T & F & T \\
        F & T & T \\
        F & F & F \\
      \end{array}
    \]
Clicker Question

- Choose the correct truth table for: $(\neg p) \lor q$

<table>
<thead>
<tr>
<th></th>
<th>A.</th>
<th></th>
<th>B.</th>
<th></th>
<th>C.</th>
<th></th>
<th>D.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>(\neg p) \lor q</td>
<td>p</td>
<td>q</td>
<td>(\neg p) \lor q</td>
<td>p</td>
<td>q</td>
<td>(\neg p) \lor q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thursday, 1/23/14, Slide #6
Truth Tables for Multiple Operators

Definition: Two propositions \( p \) and \( q \) are equivalent if they have the same truth table values.

- Notation: \( p \equiv q \)

Example 2: Construct truth tables for \( (p \lor q) \land r \) and \( p \lor (q \land r) \)

- How many rows are in the table?
- Are these two propositions equivalent?
- How should we evaluate \( p \lor q \land r \) with no parentheses?
Rules of Precedence

- These tell us how to evaluate expressions that are not made clear by parentheses:
  - 2 + 3 * 4 means 2 + (3 * 4)

- Logical Rules of Precedence:
  - **NOT** – highest precedence
  - **AND** – 2\textsuperscript{nd} highest precedence
  - **OR** – 3\textsuperscript{rd} highest precedence

- Examples:
  - How should we evaluate \( p \lor q \land r \)?
  - Use precedence rules to assign parentheses to:
    - \( \neg p \land q \lor \neg r \)

- Best practice: Use parentheses to clarify!