1. Below is a sample. For the data from this sample, compute the mean, median, mode, variance and standard deviation. [10 pts]

   1  
   4  
   6  
   5  
   3  
   5  
   2  
   5  
   3

\[ \Sigma X = 34 \]
\[ \Sigma X^2 = 150 \]

\[ \overline{X} = \frac{\sum X}{n} = \frac{34}{9} = 3.78 \]

\[ SS = \sum X^2 - \left( \frac{\sum X}{n} \right)^2 = 150 - \frac{1156}{9} = 21.56 \]

\[ s^2 = \frac{SS}{n-1} = \frac{21.56}{8} = 2.695 \]

\[ s = \sqrt{s^2} = \sqrt{2.695} = 1.64 \]

The median would be 4 (4 scores above and 4 scores below). The mode would be 5 (most frequently occurring score). Thus, the distribution would be a bit negatively skewed.

2. Test the hypothesis that the sample you see in the first question was drawn from a normal population with \( \mu = 5 \) and \( \sigma = 2 \). [10 pts]

Because \( \sigma \) is known, the appropriate statistic would be a z-score.

\[ H_0: \mu = 5 \]
\[ H_1: \mu \neq 5 \]

Decision Rule: If \( |z_{\text{obs}}| \geq 1.96 \), reject \( H_0 \).

\[ \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{9}} = .67 \]

\[ z = \frac{\overline{X} - \mu_0}{\sigma_{\overline{X}}} = \frac{3.78 - 5}{.67} = -1.82 \]
Because $|z_{\text{obs}}| < 1.96$, we would retain $H_0$. Thus, it is plausible that the sample was drawn from a population with $\mu = 5$. OTOH, you should realize that you may be making a Type II error, because with only 9 scores in your sample, you may have insufficient power.
3. You all remember the OJ Simpson trial, right? OK, tell me in words what a Type I and a Type II error would be for that trial, which tested the H₀ that Simpson was not guilty. In this context, which error would be more serious? Why? In typical psychological research, we set α = .05. What level do you think α is set to for most criminal trials in this country? Why? [10 pts]

A Type I Error would be concluding that OJ was guilty, when in fact he was innocent. A Type II Error would be concluding that OJ was innocent, when in fact he was guilty. Because we generally think that it’s a very serious error to falsely imprison (or execute) an innocent person, a Type I Error would be thought of as more serious. On that basis, courtroom decisions are likely to have a probability of Type I Error set at a “probability” that is much smaller than .05 (one would hope .001 or so), because we would not want to tolerate falsely convicting 5 people out of 100 on trial.

4. The average age for registered voters in the county is μ = 39.7 years with σ = 11.8. The distribution of ages is approximately normal. During a recent jury trial in the county courthouse, a statistician noted that the average age for the 12 jurors was \( \bar{X} = 50.4 \) years. [10 pts]
   a. How likely is it to obtain a jury this old or older by chance?

   \[
   \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{11.8}{\sqrt{12}} = 3.41 \\
   z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{50.4 - 39.7}{3.41} = 3.14
   \]

   The probability of a z of 3.14 or greater is .0008.

   b. Is it reasonable to conclude that this jury is not a random sample of registered voters?

   Yes, it is extremely unlikely to draw a sample of 12 from a population with μ = 39.7 and have the sample mean turn out to be 50.4. (Of course, it is possible to get such a sample from that population, just very unlikely! So, you could be making a Type I Error.)

5. As we saw in class, gestation periods are normally distributed with μ = 268 days and σ = 16 days. Suppose that 16 women became pregnant. How likely is it that the mean gestation period for this sample would fall between 276 and 284 days? [5 pts]

   \[
   \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{16}} = 4 \\
   z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{276 - 268}{4} = 2 \\
   z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{284 - 268}{4} = 4
   \]

   The proportion of scores greater than or equal to z=2 would be .0228. The proportion of scores greater than or equal to z=4 would be .00003. Thus, the proportion between z=2 and z=4 would be .02277, or roughly 2.3%.
6. A normal distribution has a mean of 120 and a standard deviation of 20. For this distribution, [5 pts]

a. What score separates the top 40% (highest scores) from the rest?

.40 in Column C yields a \( z = .25 \), which would mean a score = 125.

b. Scores between 60 and 100 make up what percentage of the distribution?

\[
\begin{align*}
  z &= \frac{X - \mu}{\sigma} = \frac{60 - 120}{20} = -3 \\
  z &= \frac{X - \mu}{\sigma} = \frac{100 - 120}{20} = -1
\end{align*}
\]

The proportion of scores less than \( z = -1 \) would be .1587. The proportion of scores less than \( z = -3 \) would be .0013. Thus, the proportion of scores between 60 and 100 would be .1587 - .0013 = .1574.

c. What range of scores would form the middle 60% of this distribution?

The middle 60% of the distribution means that you're eliminating the extreme 20% of the scores in each end, so you'd look up .20 in Column C. That yields \( z = 0.84 \), so -0.84 and +0.84 would cut off the lower and upper 20% of the distribution (leaving the middle 60%). Thus, the values that would cut off the middle 60% of the distribution would be 103.2 and 136.8.