

Chapter 9: Introduction to the t Statistic

First of all, you need to know who developed the t statistic. His name was William S. Gossett, but he published under the pseudonym *Student*. (His employer wouldn't let him publish under his own name.) Thus, the statistic is often called Student's t . But that's not the good part. Gossett actually had the world's best job...even better than L'il Abner's job as a mattress tester. Can you guess how he earned his living?

You've learned how to test hypotheses about means using z -scores, but you should also recognize that the test is fairly artificial, because it requires that you know σ in order to compute $\sigma_{\bar{X}}$:

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

Even though you would rarely know σ , you could estimate it by using the sample standard deviation. When you must use s to estimate σ , however, you can no longer compute $\sigma_{\bar{X}}$. Instead, you will now compute an estimated standard error:

$$s_{\bar{X}} = \hat{\sigma}_{\bar{X}} = \sqrt{s^2/n} = s/\sqrt{n}$$

When you create a standard score using the estimated standard error, you will have created a t statistic:

$$t = \frac{\bar{X} - \mu}{s_{\bar{X}}}$$

The t statistic is used to test hypotheses about an unknown population mean μ in situations where the value of σ is unknown. The formula for the t statistic has the usual standard score structure, and thus closely resembles the z -score, but with the estimated standard error in the denominator.

Unlike the unit normal distribution, there is not a single t distribution. Instead there is a family of t distributions, determined by the degrees of freedom found in the denominator of the sample variance ($n - 1$). The t distributions are unimodal and symmetrical, with a mean of 0. However, the tails are typically higher than those found in a normal distribution, except for very large sample sizes (approaching infinity), in which case the t distribution is normal. As a result, the values of t that cut off the upper and lower 2.5% of the distribution (using the typical α -level and a two-tailed test) will vary. In order to determine those values, a new table is needed. I've highlighted the column that you will use exclusively ($\alpha = .05$, two-tailed).

TABLE B.2 THE t DISTRIBUTION

df	PROPORTION IN ONE TAIL					
	0.25	0.10	0.05	0.025	0.01	0.005
	PROPORTION IN TWO TAILS					
	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
∞	0.674	1.282	1.645	1.960	2.326	2.576

Note that the t -distribution approaches the normal distribution with large sample size, because the critical value of t for an infinite sample size is 1.96, which is the z -score that would cut off 2.5% of the normal distribution.

Hypothesis Testing with the t Statistic

G&W provide an example using an experiment based on Scaife's (1976) study of aversion to eye-spot patterns among insectivorous birds. Birds are placed in a cage, which is divided in half. One side has eye spots and the other side does not. If the birds don't try to avoid the eye spots, then they will stay in each half of the cage for an equal duration (the dv). Birds are placed in the cage for an hour, so they should spend about 30 minutes on each side of the cage.

$$H_0: \mu_{\text{Plain Side}} = 30$$

$$H_1: \mu_{\text{Plain Side}} \neq 30$$

$$\alpha = .05$$

$$n = 16, \text{ so } t_{\text{Critical}}(15) = 2.131$$

Decision rule: If $|t_{\text{Obtained}}| \geq 2.131$, reject H_0 .

For this sample of 16 birds, the mean time spent on the plain side (\bar{X}) was 39 minutes. With $SS = 540$, you can compute the necessary values as follows:

$$SS = 540$$

$$s^2 = \frac{540}{15} = 36$$

$$s_{\bar{X}} = \sqrt{s^2/n} = \sqrt{36/16} = 1.5$$

You can now compute the t statistic as follows:

$$t = \frac{39 - 30}{1.5} = 6$$

You would reject H_0 , because $|t_{\text{Obtained}}| \geq t_{\text{Critical}}$

You could then conclude that the birds spend significantly more time on the plain side of the cage than on the side with the eye spots.

You might report your results as follows:

The birds spent more time on the plain side of the apparatus ($M = 39$, $SD = 6$), $t(15) = 6.0$, $p < .05$.

Effect Size

Effect size could be measured with Cohen's d as follows:

$$d = \frac{\text{mean difference}}{\text{sample standard deviation}} = \frac{9}{6} = 1.5$$

This effect size would be quite large. (As a rough guide, $d = .2$ or less is a small effect size, a d between $.2$ and $.8$ is a medium effect size, and a d greater than $.8$ is a large effect size.)

Another measure of effect size is r^2 :

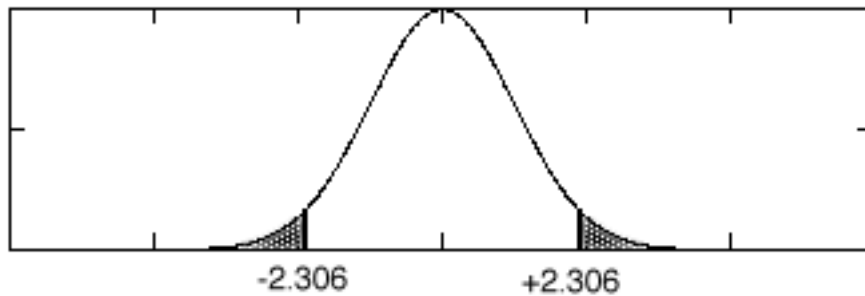
$$r^2 = \frac{t^2}{t^2 + df} = \frac{6^2}{6^2 + 15} = .71$$

For r^2 , a small effect is between $.01$ and $.09$, a medium effect is between $.09$ and $.25$, and a large effect is $.25$ or greater. Thus, using either d or r^2 , this effect is quite large.

QUESTION: Do women's salaries differ from men's? Assume that we know that the mean salary for men is \$41k (2003 census, \$40,668).

STATISTICAL HYPOTHESES: $H_0: \mu = \$41k$
 $H_1: \mu \neq \$41k$

DECISION RULE: Reject H_0 if t_{obt} occurs with $p < .05$.
With a sample $n = 9$, $df = 8$, $t_{crit} = 2.306$.



PICTURE:

DATA:

28
20
35
37
39
30
31
29
28

FORMULAS: $t_{obt} = \frac{\bar{X} - \mu}{S_{\bar{X}}}$ $S_{\bar{X}} = \frac{s}{\sqrt{n}}$ $d = \frac{\bar{X} - \mu}{s}$

DECISION:

CONCLUSION/INTERPRETATION:

A sample of 25 reaction time scores was selected from a group of professional athletes to see if their reflexes were faster than those found in the normal population. The sample had a mean of 10 sec and a sum of squares of 600. On the basis of years of study of normal, randomly selected people, we know that the distribution is normal with μ of 13 sec. How likely is it that the sample was obtained from the normal population?

How would your computations differ if you were told that the sample size was 100?

If you were told that the standard deviation of the population (σ) was 2, what would you do differently from part 1?

When individuals feel disconnected from others in society, we say that they feel alienated. Dr. Conrad, a clinical psychologist, measures alienation with a survey scale that consists of 20 items, where the higher the score, the greater the feeling of alienation. The scale is normalized so that a score of 40 or below is considered healthy and not alienated. For 25 patients who are seeking care for anxiety, he finds a mean alienation score of 46 with a standard deviation of 13.2. Test the hypothesis that the sample was drawn from a population of people with a healthy alienation score of 40. [Ritchey, 332]

You have a measurement index of happiness with scores ranging from 1 to 10. You wish to test the hypothesis that the mean score of the Elmsville area is 6.5. You draw a random sample of 59 individuals and find a mean score of 5.9 with a standard deviation of 2.6. Test the hypothesis. [Ritchey, 332]

On a standardized spatial skills task, normative data reveal that people typically get $\mu = 15$ correct solutions. A psychologist tests $n = 7$ individuals who have brain injuries in the right cerebral hemisphere. For the following data, determine whether or not right-hemisphere damage results in significantly different performance on the spatial skills task.

	X	X ²
	12	144
	16	256
	9	81
	8	64
	10	100
	17	289
	10	100
Sum	82	1034

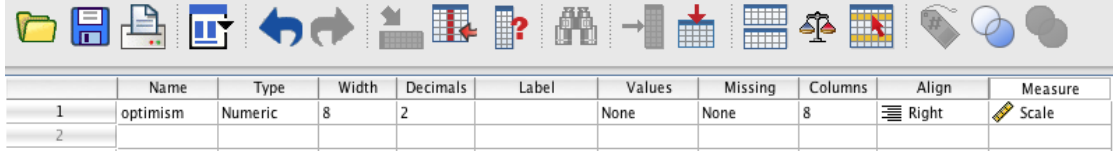
A psychologist is interested in the long-term effects of a divorce on the children in a family. A sample is obtained of $n = 10$ children whose parents were divorced at least five years ago. Each child is given a personality questionnaire measuring depression. In the general population, the scores on this questionnaire form a normal distribution with $\mu = 80$ and $\sigma = 12$. The scores for this sample are seen below. Do children from divorced families appear to differ in levels of depression from children in the general population?

	X	X ²
	83	6889
	81	6561
	75	5625
	92	8464
	84	7056
	107	11449
	63	3969
	112	12544
	92	8464
	88	7744
Sum	877	78765

Using SPSS to Conduct Single-Sample t-Tests

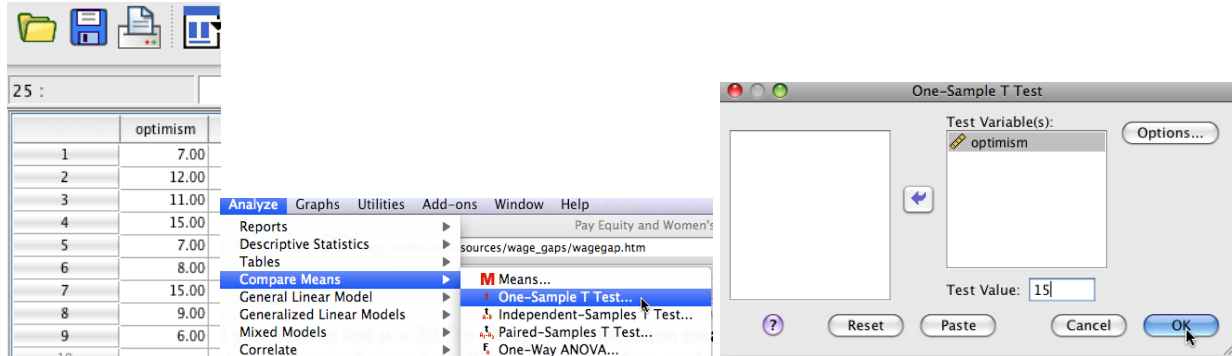
Single-Sample t -Test problem from Gravetter & Wallnau, 9th Edition (Demo 9.1)

First set up the data (in Variable View), as seen below:



	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure
1	optimism	Numeric	8	2		None	None	8	Right	Scale
2										

Next, enter the data (in Data View) as seen below left:



The Data View shows the following data for the 'optimism' variable:

	optimism
1	7.00
2	12.00
3	11.00
4	15.00
5	7.00
6	8.00
7	15.00
8	9.00
9	6.00

The One-Sample T Test dialog box shows the 'Test Variable(s):' field containing 'optimism' and the 'Test Value:' field set to 15. The 'OK' button is highlighted.

Once the data are entered, click-and-drag the **Analyze** menu to **Compare Means**, and then **One-Sample T Test**. You'll then see a window like the one above on the right. Click-and-drag the **optimism** variable from the left into the box on the right. Note that to test $H_0: \mu = 15$, I've entered 15 into the *Test Value* window. Click on OK when you have done so and you will see an analysis window opened. The analysis should appear, as seen below. Note that there is no need to use the table of critical values of t , because the program shows the exact probability of obtaining a t of -4.376 (in this case $.002$). All you have to do is determine if the probability level is greater than $.05$ (retain H_0) or less than $.05$ (reject H_0). In this case, you would reject $H_0: \mu = 15$ and conclude that the optimism scores were significantly lower in this class.

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
optimism	9	10.0000	3.42783	1.14261

One-Sample Test

	Test Value = 15					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
optimism	-4.376	8	.002	-5.00000	-7.6349	-2.3651

A sample of quiz scores was obtained from a statistics class. The SPSS analysis conducted on these data is seen below.

	N	Mean	Std. Deviation	Std. Error Mean
Quiz Scores	36	7.4444	1.36161	.22693

	Test Value = 8					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Quiz Scores	-2.448	35	.020	-.55556	-1.0163	-.0949

- State the null and alternative hypothesis.
- How many quiz scores are involved in this analysis?
- What is your decision about H_0 ? What should you conclude?
- How would you describe the effect size of this analysis?